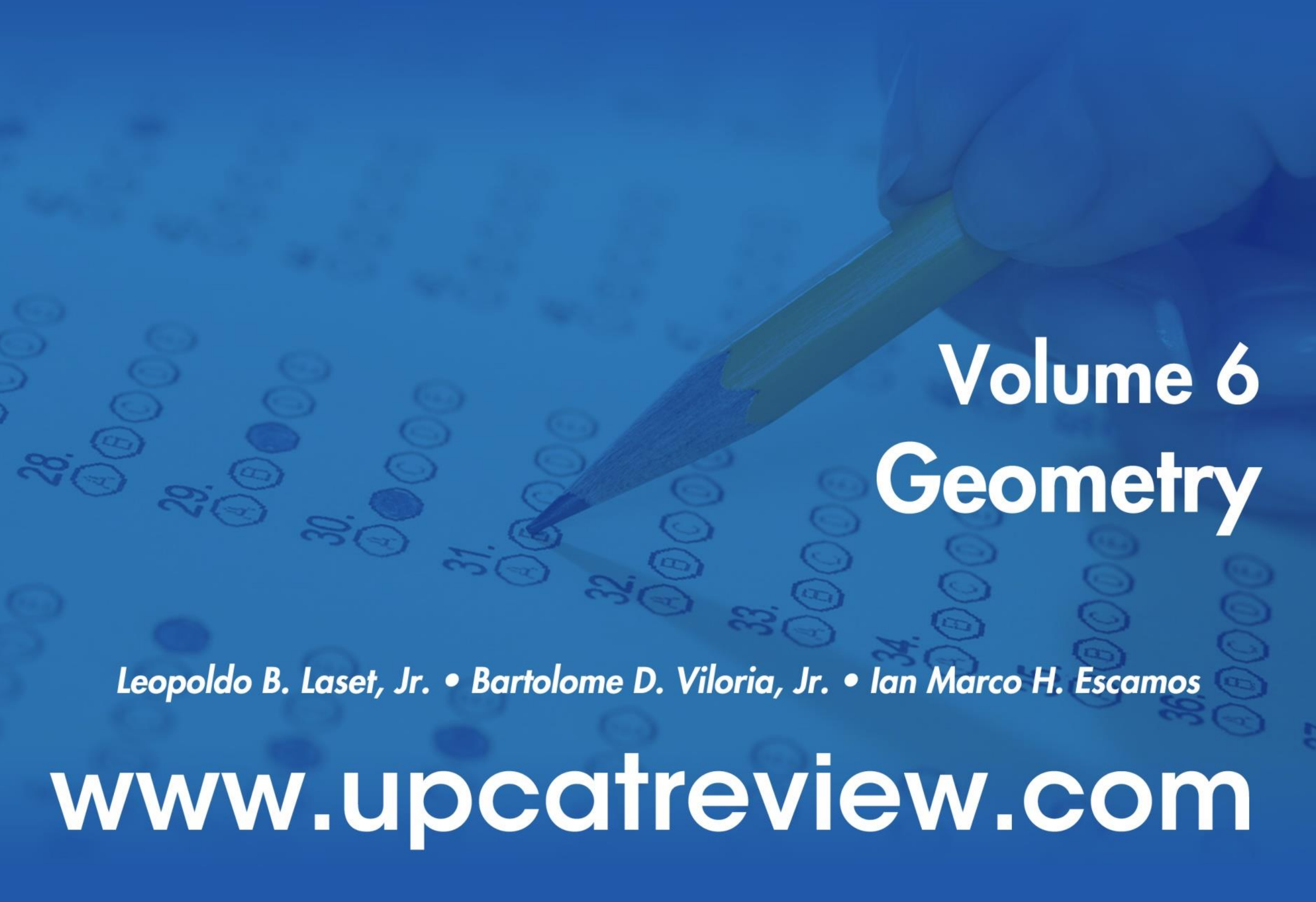
review masters

UPCAT Review

Compiled UPCAT Questions



Volume 6
Geometry

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UPCAT Review – Volume 6 – Geometry
Downloadable e-Book

PREFACE

Believe That You Can Pass the UPCAT!

by Leopold Laset

Do you sometimes find it hard to believe that your dream to pass the UPCAT can become a reality? If so, then there is something very important that you need to know.

UPCAT is for dreamers like you.

Every student who passed the UPCAT began thinking or dreaming of passing the UPCAT.

Your near-perfect or perfect score in a quarterly test, your cellphone, PSP, or any gadget, your out-of-town (or out-of-country) vacation, your new pair of shoes, and any other stuff that you desired and now possess - are all the result of your 'dream come true'.

What this means is that throughout your lifetime, you have had an idea, you have desired for many things and worked hard for them, overcome problems and ultimately transformed your dream into reality.

And if hundreds and thousands of students have been able to pass the UPCAT in the past, by starting with a dream, then it stands to reason, that you can do it too.

Often we make the mistake of thinking that UPCAT is for a small number of bright students who have the brains and intelligence that we don't possess.

But this is simply not true.

The fact that thousands of average students have brought their dreams of passing the UPCAT to fruition in the past demonstrates that the opportunity to qualify in the UPCAT is something that is available to each UPCAT aspirant – average or bright.

Right now, hundreds of UPCAT dreamers are taking the steps necessary to achieve the goals of passing the UPCAT. Some are studying this early, some are joining community of fellow dreamers, and some are attending review classes. What is it that you need to do?

In order to achieve your goal of passing the UPCAT, the only things you really need are:

(1) A crystal clear picture that you already passed the UPCAT
(2) An unshakeable determination to do whatever it takes to make your dream of passing the UPCAT a reality

As soon as you take these two steps, passing the UPCAT becomes achievable. If you need a help – you look for it. If you encounter a difficult concept – you find a way to understand it. If you can't solve a math problem – you try and try and practice more.

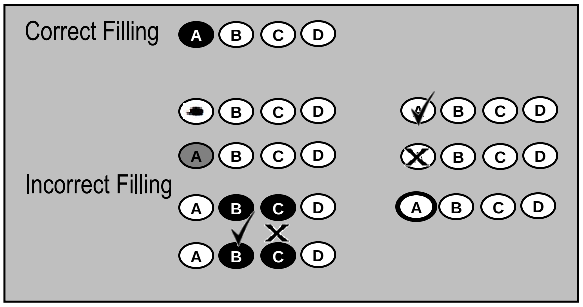
And gradually, step-by-step, you bring your UPCAT dream into reality to join the dreams of the thousands of UPCAT dreamers who have gone before you.

So today I'd like to encourage you to believe in yourself and appreciate the fact that you live in a world where 'dreams do come true'.

Understand that thousands of students have made their UPCAT dream a reality in the past – Thousands more will make their UPCAT dream a reality in the near future and you CAN be one of them.

TABLE OF CONTENTS

ANSWER SHEET – GEOMETRY



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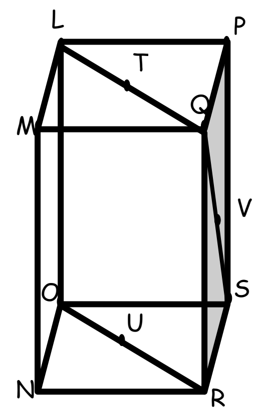
1.	A B C D	26.	A B C D	51.	A B C D	76.	A B C D
2.	A B C D	27.	A B C D	52.	A B C D	77.	A B C D
3.	A B C D	28.	A B C D	53.	A B C D	78.	A B C D
4.	A B C D	29.	A B C D	54.	A B C D	79.	A B C D
5.	A B C D	30.	A B C D	55.	A B C D	80.	A B C D
6.	A B C D	31.	A B C D	56.	A B C D	81.	A B C D
7.	A B C D	32.	A B C D	57.	A B C D	82.	A B C D
8.	A B C D	33.	A B C D	58.	A B C D	83.	A B C D
9.	A B C D	34.	A B C D	59.	A B C D	84.	A B C D
10.	A B C D	35.	A B C D	60.	A B C D	85.	A B C D
11.	A B C D	36.	A B C D	61.	A B C D	86.	A B C D
12.	A B C D	37.	A B C D	62.	A B C D	87.	A B C D
13.	A B C D	38.	A B C D	63.	A B C D	88.	A B C D
14.	A B C D	39.	A B C D	64.	A B C D	89.	A B C D
15.	A B C D	40.	A B C D	65.	A B C D	90.	A B C D
16.	A B C D	41.	A B C D	66.	A B C D	91.	A B C D
17.	A B C D	42.	A B C D	67.	A B C D	92.	A B C D
18.	A B C D	43.	A B C D	68.	A B C D	93.	A B C D
19.	A B C D	44.	A B C D	69.	A B C D	94.	A B C D
20.	A B C D	45.	A B C D	70.	A B C D	95.	A B C D
21.	A B C D	46.	A B C D	71.	A B C D	96.	A B C D
22.	A B C D	47.	A B C D	72.	A B C D	97.	A B C D
23.	A B C D	48.	A B C D	73.	A B C D	98.	A B C D
24.	A B C D	49.	A B C D	74.	A B C D	99.	A B C D
25.	A B C D	50.	A B C D	75.	A B C D	100.	A B C D

CHAPTER 1: Basic Ideas of Geometry

For Numbers 1 to 4, refer to the figure to the right:



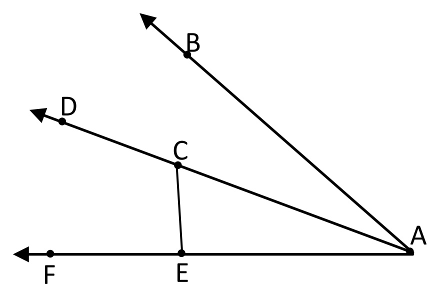
1. Which of the following is a set of collinear points?
A. M, L, P
B. Q, R, S
C. O, U, R
D. S, V, P

2. V is between points ______.
A. Q and S
B. R and S
C. P and S
D. Q and P

3. S is the intersection of ______.
A. $\overleftrightarrow{QV}$ and $\overleftrightarrow{PS}$
B. $\overleftrightarrow{RS}$ and $\overleftrightarrow{PS}$
C. $\overleftrightarrow{RS}$ and $\overleftrightarrow{QV}$
D. all of these

4. Which of the following is a set of coplanar lines?
A. $\overleftrightarrow{LM}, \overleftrightarrow{LP}$ and $\overleftrightarrow{LO}$
B. $\overleftrightarrow{ON}, \overleftrightarrow{OS}$ and $\overleftrightarrow{OR}$
C. $\overleftrightarrow{SQ}, \overleftrightarrow{SO}$ and $\overleftrightarrow{SR}$
D. $\overleftrightarrow{PQ}, \overleftrightarrow{PS}$ and $\overleftrightarrow{PL}$

For Numbers 5 to 7, refer to the figure to the right:



5. E is the midpoint of $\overline{AF}$ if ______.
A. EF = EA
B. EF = ½AF
C. AF = 2·EA.
D. All of these

6. Which of the following is true?
A. AC + CE = AE
B. FE + EC = FC
C. m∠ACE + m∠CEA = m∠EAC
D. m∠BAD + m∠DAF = m∠BAF

7. If C is the midpoint of $\overline{AD}$, AD = 4y + 8, and AC = 5y – 8, what is DC?
A. 4
B. 8
C. 12
D. 24

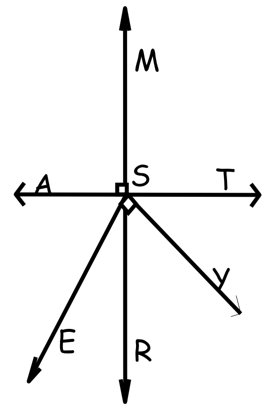
8. What is the inverse of the statement: "If there's a will, then there's a way."?
A. If there's no way, then there's no will.
B. If there's no will, then there's no way.
C. If there's a way, then there's a will.
D. If there's a way, then there's no will.

9. What type of triangle is the one with vertex coordinates (0, 4), (3, 0) and (3, 4)?
A. scalene
B. isosceles
C. equilateral
D. equiangular

10. If a triangle is equilateral, then it is equiangular. △XYZ is not equiangular. What conclusion can you draw?
A. △XYZ is not equilateral.
B. △XYZ is not isosceles.
C. △XYZ is not scalene.
D. △XYZ is not acute.

CHAPTER 2: Introduction to Proof

For Numbers 11 to 14, refer to the figure to the right:



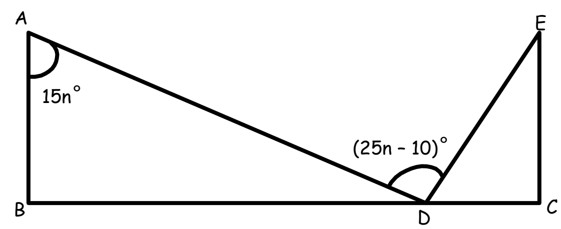
11. Which of the following are complementary angles?
A. ∠MSA and ∠MST
B. ∠ASE and ∠RSY
C. ∠ESR and ∠RSY
D. ∠ESR and ∠TSY

12. Which of the following are supplementary angles?
A. ∠MSA and ∠ESY
B. ∠MST and ∠ASE
C. ∠ASE and ∠TSY
D. ∠ESR and ∠RSY

13. Which of the following are congruent angles?
A. ∠RST and ∠ESY
B. ∠RSA and ∠MSA
C. ∠ASE and ∠RSY
D. all of these

14. Which of the following are perpendicular?
A. $\overrightarrow{SM}$ & $\overrightarrow{SY}$ B. $\overrightarrow{SA}$ & $\overrightarrow{SR}$ C. $\overrightarrow{SR}$ & $\overrightarrow{SM}$ D. $\overrightarrow{ST}$ & $\overrightarrow{SE}$

For Numbers 15 to 17, refer to the figure to the right:



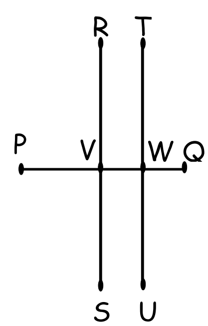
Given: $\overline{AD} \perp \overline{DE}$

15. m∠ADE = ________.
A. 74
B. 90
C. 99
D. 110

16. m∠BAD = ________.
A. 30 B. 45 C. 60 D. 75

17. m∠CED = ________.
A. 30 B. 45 C. 60 D. 75

For Numbers 18 to 20, refer to the figure to the right:



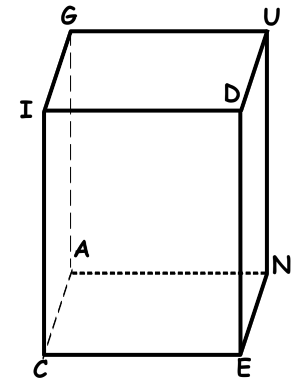
18. If $\overline{RS}$ is the perpendicular bisector of $\overline{PQ}$, then ______.
A. RV = VS
B. $\overline{TU} \perp \overline{PQ}$
C. PV = VQ
D. TW = WU

19. If VW = WQ and $\overline{TU} \perp \overline{PQ}$, then ______.
A. $\overline{PQ}$ is the perpendicular bisector of $\overline{TU}$
B. $\overline{VQ}$ is the perpendicular bisector of $\overline{TU}$
C. $\overline{TU}$ is the perpendicular bisector of $\overline{PQ}$
D. $\overline{TU}$ is the perpendicular bisector of $\overline{VQ}$

20. If $\overline{PQ}$ is the perpendicular bisector of $\overline{RS}$, then the ratio of RS to RV is ________.
A. 4:1 B. 3:1 C. 2:1 D. 1:2

CHAPTER 3: Parallel Lines and Planes

For Numbers 21 to 24, refer to the rectangular prism to the right:



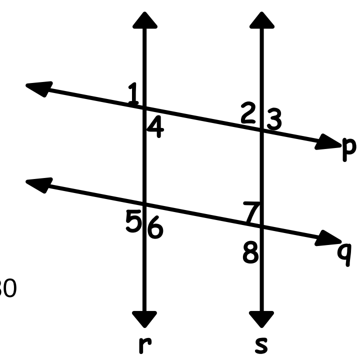
21. How many sides of the rectangular prism are parallel to $\overline{GA}$?
A. 4 B. 3 C. 2 D. 1

22. How many sides of the rectangular prism are skew to $\overline{GA}$?
A. 7 B. 6 C. 5 D. 4

23. Which face of the rectangular prism is parallel to IDCE?
A. DUNE C. GACI
B. UGAN D. CANE

24. Which line is parallel to $\overleftrightarrow{CN}$?
A. $\overleftrightarrow{IU}$ B. $\overleftrightarrow{GD}$ C. $\overleftrightarrow{GU}$ D. $\overleftrightarrow{ID}$

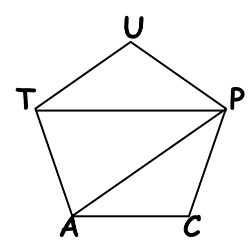
For Numbers 25 to 27, refer to the figure to the right:



25. If p || q, then ______.
A. $\angle 1 \cong \angle 5$ C. $\angle 3 \cong \angle 7$
B. $\angle 4 \cong \angle 6$ D. $\angle 4 \cong \angle 7$

26. If $m\angle 6 + m\angle 8 = 180$, then ______.
A. r || s C. $m\angle 6 + m\angle 7 = 180$
B. p || q D. $m\angle 6 = m\angle 4$

27. If $r \perp p$, $r \perp q$ and r || s, then
A. $\angle 1 \cong \angle 8$ C. Both A and B
B. $m\angle 1 + m\angle 8 = 180$ D. Neither A nor B



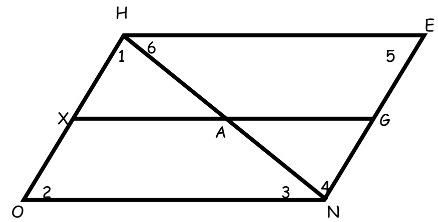
28. Given: $\overline{UT} \cong \overline{UP}$. $\overline{PT} \cong \overline{PA}$. $\angle U \cong \angle C \cong \angle UTA$. $m\angle PAT = 72^\circ$. Find: $m\angle C$.
A. 72 C. 126
B. 108 D. 144

29. Find the sum of the exterior angles of a polygon with 18 sides.
A. 360 B. 540 C. 720 D. 900

30. If the sum of the measures of five interior angles of a hexagon is 640°, what is the measure of the sixth interior angle?
A. 90 B. 80 C. 70 D. 60

CHAPTER 4: Congruent Triangles

For Numbers 31 to 34, refer to the figure to the right:



31. If △OHN ≅ △ENH, then ______.
A. ∠1 ≅ ∠4 C. ∠3 ≅ ∠5
B. ∠2 ≅ ∠6 D. $\overline{HX} \cong \overline{NG}$

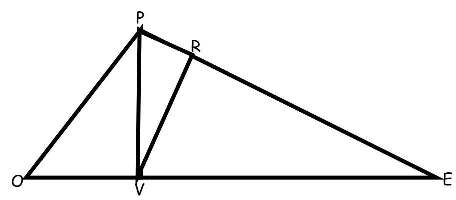
32. If $\overline{OH} \cong \overline{HN}$, then ______.
A. ∠2 ≅ ∠5 B. $\overline{OH} \cong \overline{NE}$ C. $\overline{HN} \cong \overline{NE}$ D. ∠2 ≅ ∠3

33. If △HXA ≅ △NGA, then ______.

A. $\overline{OX} \cong \overline{EG}$ B. $\overline{AN} \cong \overline{XH}$ C. $\overline{ON} \cong \overline{EH}$ D. $\overline{XA} \cong \overline{GA}$

34. If A is the midpoint of both $\overline{HN}$ and $\overline{XG}$, then ______.
A. △OHN ≅ △ENH B. △XHA ≅ △GNA C. Both A & B D. Neither A nor B

For Numbers 35 to 37, refer to the figure to the right:

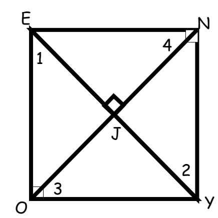


35. If $\overline{PV}$ is the perpendicular bisector of $\overline{OE}$, then ______.
A. V is the midpoint of $\overline{OE}$
B. $\overline{PV}$ is a median of △OPE
C. Both A and B
D. Neither nor B

36. If $\overline{VR} \perp \overline{PE}$, then $\overline{VR}$ is an altitude of both ______.
A. △POV and △PEV C. △PRV and △POE
B. △POV and △PRV D. △ERV and △PEV

37. If △OPV ≅ △EPV, then ______.
A. △ OPE is isosceles C. ∠POV ≅ ∠PEV
B. $\overline{PV} \perp \overline{OE}$ D. all of these

For Numbers 38 to 40, refer to the following figure:



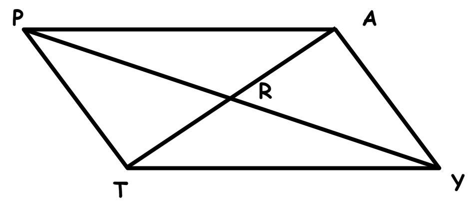
38. If ∠1 ≅ ∠2, then △EOY≅ △YNE by ______.
A. LL Congruence C. HL Congruence
B. LA Congruence D. HA Congruence

39. If ∠3 ≅ ∠4 and $\overline{OJ} \cong \overline{NJ}$, then △OJY ≅ △NJE by ______.
A. LL Congruence C. HL Congruence
B. LA Congruence D. HA Congruence

40. If $\overline{EN} \parallel \overline{YO}$ and $\overline{EN} \cong \overline{YO}$, then △ONE ≅ △NOY by ______.
A. SSS Congruence C. ASA Congruence
B. SAS Congruence D. SSA Congruence

CHAPTER 5: Using Congruent Triangles and Parallel Lines

For Numbers 41 to 44, refer to the figure to the right:



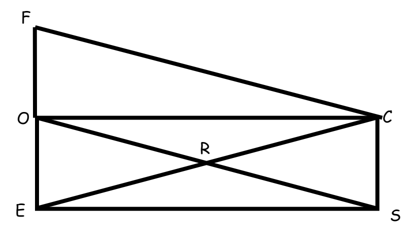
41. If PAYT is a parallelogram, then ______.
A. PR = YR and TR = AR
B. PA = YT and PT = YA
C. m∠APT = m∠TYA and m∠PAY = m∠YTP
D. All of these

42. If AT = PY and $\overline{AT} \perp \overline{PY}$, then PAYT is a ______.
A. parallelogram B. rhombus C. square D. rectangle

43. If PAYT is a rhombus, m∠RPA = 35, then m∠RAP = ______.
A. 145 B. 65 C. 55 D. 35

44. If PAYT is a rectangle, then ______.
A. $\overline{AT} \perp \overline{PY}$ B. AT = PY C. m∠PAT = m∠YAT D. AP = RT

For Numbers 45 to 47, refer to the figure to the right:



Given: FOSC is a parallelogram and OCSE is a rectangle.

45. △FCE is a/an ______ triangle
A. right
B. scalene
C. isosceles
D. equilateral

46. If OE = OC, then ______.
A. △COE is a 45°-45°-90° △
B. OCSE is a square
C. both A and B
D. neither A nor B

47. Which of the following pairs of triangles are congruent?
A. △OCR and △SER
B. △ORE and △SRC
C. △FOC and △CSE
D. All of these

48. Which of the following is NEVER true?
A. The legs of a trapezoid are congruent.
B. Opposite angles of a trapezoid are supplementary.
C. Two angles of a trapezoid are right.
D. The bases of a trapezoid are congruent.

49. Which of the following properties is common to an isosceles trapezoid and a rectangle?
A. The diagonals are congruent.
B. Opposite angles are congruent.
C. Opposite sides are congruent.
D. Consecutive angles are congruent.

50. If at least two midsegments of a triangle are congruent, then the triangle is ______.
A. scalene
B. right
C. isosceles
D. equilateral

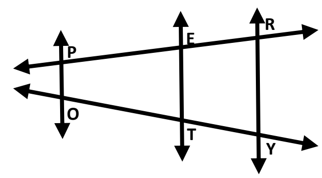
CHAPTER 6: Similarity

51. Which of the following pairs of triangles are similar?
A. Two isosceles right triangles
B. Two triangles with a common base
C. Two overlapping triangles
D. Two scalene triangles

52. Which of the following pairs of polygons are similar?
A. Two right triangles
B. Two rectangles
C. Two regular pentagons
D. Two kites

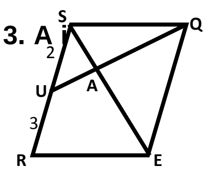
53. If $\frac{y}{5} = \frac{3}{7}$, then ______.
A. $\frac{y}{5} = \frac{7}{3}$
B. $\frac{5}{y} = \frac{3}{7}$
C. $\frac{y+5}{5} = \frac{8}{7}$
D. 7y = 15

54. Leo runs 2 kilometers in 15 minutes. At this rate how long will it take him to run a 25-km marathon?
A. 2 hrs and 22.5 min
B. 2 hrs and 30 min
C. 2 hrs and 45 min
D. 3 hrs and 7.5 min



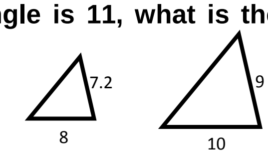
55. In the figure, $\overleftrightarrow{PO} \parallel \overleftrightarrow{ET} \parallel \overleftrightarrow{RY}$, PE = 8, ER = 4 and OT = 10, then TY = ______.
A. 4
B. 5
C. 6
D. 7

56. SQER is a parallelogram and U is a point on the side SR such that SU = 2 and UR = 3. A is the point of intersection of SE and UQ. The ratio of SA:AE is equal to ______.



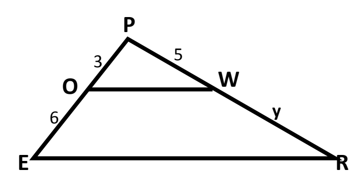
A. 2:3
B. 1:2
C. 1:3
D. 2:5

57. The two triangles are similar. If the length of the missing side of the larger triangle is 11, what is the length of the missing side of the smaller triangle?



A. 13.75
B. 10.2
C. 8.8
D. 7.6

58. What must be the value of y in order that $\overline{OW}$ be parallel to $\overline{ER}$?



A. 8
B. 10
C. 12
D. 15

59. If the sides of a quadrilateral are 8, 14, 12, and 20 cm and the longest side of a similar quadrilateral is 30 cm, how long is the shortest side of this quadrilateral?
A. 6 cm
B. 10 cm
C. 12 cm
D. 14 cm

60. What is the geometric mean between 18 and 72?
A. 36
B. $24\sqrt{2}$
C. $48\sqrt{2}$
D. 45

CHAPTER 7: Right Triangles

61. Which of the following is NOT true about right triangles?

A. The longest side is the longer leg.
C. The legs are perpendicular.
B. The hypotenuse is opposite 90°.
D. The acute angles are complementary.

62. If the sum of the squares of the lengths of the two shorter sides of a triangle is less than the square of the length of the longest side, the triangle is ______.

A. acute B. obtuse C. scalene D. right

63. Which of the following sets of numbers forms a Pythagorean Triple?

A. {1, 2, 3}
C. {9, 16, 25}
B. {20, 21, 29}
D. $\{\sqrt{3}, \sqrt{4}, \sqrt{7}\}$

64. Two cars start at the same time and in the same place. One travels due north at 60 kph while the other due east at 80 kph. How far are they from each other after one hour?

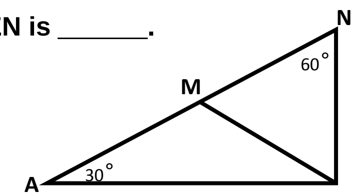
A. 140 km B. 100 km C. 70 km D. 50 km

65. Which of the following are the sides of an acute triangle?

A. 6, 8, 9 B. 15, 20, 27 C. 21, 28, 35 D. 27, 36, 48

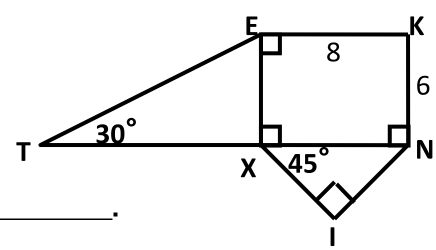
66. Given △ANE. If $\overline{EM}$ is an altitude, then △MEN is ______.

A. an equilateral triangle
B. an isosceles triangle
C. a right triangle
D. an obtuse triangle



67. Find the area of hexagon *TEKNIX*.

A. $26 + 6\sqrt{3} + 8\sqrt{2}$
C. $64 + 18\sqrt{3}$
B. 73
D. $26 + 14\sqrt{6}$

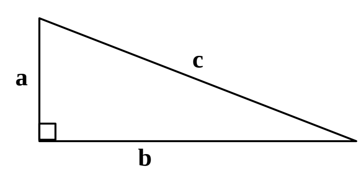


68. In a right triangle, as an acute angle gets smaller, ______________________.

A. the cosine gets closer to 1 and the sine gets closer to 0
B. the sine gets closer to 1 and the cosine gets closer to 0
C. the cosine gets closer to 0 and the tangent gets closer to 1
D. the tangent gets closer to 0 and the sine gets closer to 1

69. In the figure the right triangle has side lengths a, b, and c, where c is the length of the hypotenuse. a^2 can be found by

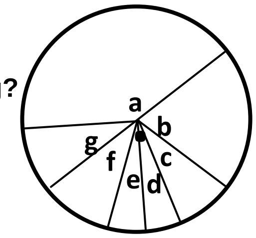
A. adding b^2 and c^2.
B. squaring the sum of b and c
C. Both A and B
D. Neither A nor B



70. Which of the following is equal to sin θ?

A. (tan θ)(cos θ) B. tan θ/cos θ C. cos θ/tan θ D. 1/cos θ

CHAPTER 8: Circles

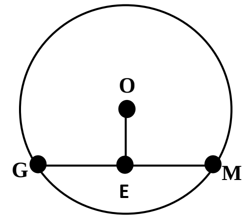


71. If a + b = 200°, and c + d + e + f = 140°, what is the number of degrees in angle g?

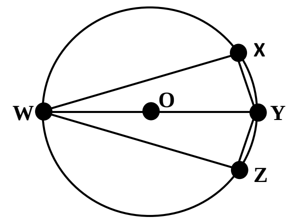
A. 10° C. 30°
B. 20° D. 45°

72. Given the circle O in the figure. If $\overline{OE} \perp \overline{GM}$, then

A. $\overline{EO} \cong \overline{GM}$ C. $\overline{EO} \cong \overline{OM}$
B. $\overline{EO} \cong \overline{OG}$ D. $\overline{GE} \cong \overline{EM}$



For Numbers 73 and 74, refer to the figure to the right:



73. If measure of arc XZ = 100, then

A. m∠XWZ = 100 C. m∠XWY = 50
B. m∠XWZ = 50 D. m∠ZWY = 50

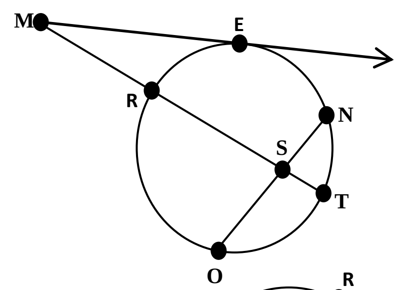
74. Which of the following is ALWAYS true?

A. m∠XWZ = 90 C. m∠XWY = 90
B. m∠WXY = 90 D. m∠XYW = 90

75. A circle with center (5, -12) is tangent to the *y*-axis in the standard (*x*, *y*) coordinate plane. What is the radius of this circle?

A. 13 B. 12 C. 7 D. 5

For Numbers 76 and 77, refer to the figure to the right:



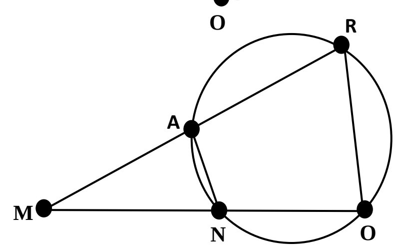
76. If measure of arc RN = 110 and measure of arc OT = 60, then

A. m∠RSN = 170 C. m∠TSO = 50
B. m∠RSN = 85 D. m∠TSO = 25

77. If measure of arc ET = 100 and measure of arc RE = 50, then

A. m∠EMR = 150 C. m∠EMR = 50
B. m∠EMR = 75 D. m∠EMR = 25

For Numbers 78 to 80, refer to the figure to the right:



78. Which angles of quadrilateral ARON are supplementary?

A. ∠RON and ∠RAN C. Both A and B
B. ∠ANO and ∠ARO D. Neither A nor B

79. If measure of arc RO = 90 and measure of arc AN = 30, then m∠RMO = _______.

A. 10 B. 20 C. 30 D. 40

80. If $\overline{NA} \perp \overline{MR}$, then ___________.

A. m∠MAN > m∠RAN C. m∠MAN = m∠ARO
B. m∠MAN = m∠RON D. m∠ARO = m∠ANO

CHAPTER 9: Area and Perimeter of Polygons

81. The area of a square is 225 sq cm. How long is one side of the square?

A. 15 cm B. 20 cm C. 25 cm D. 56.25 cm

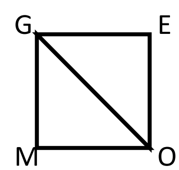
82. A rectangle is thrice as long as it is wide. If the length of the rectangle is 9 inches, what is the rectangle's perimeter, in inches?

A. 12 B. 24 C. 36 D. 72

83. A circular chip has a radius of $\frac{5}{6}$ cm. When lying flat, how much area does the coin cover, in square cm?

A. $5\pi/6$ B. $5\pi/3$ C. $25\pi/9$ D. $25\pi/36$

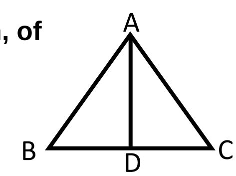
84. Square *GEOM* has a perimeter of 52 inches. How many inches long is diagonal $\overline{GO}$?



A. 26 C. $13\sqrt{3}$

B. $13\sqrt{2}$ D. 13

85. In the figure, $\overline{AB} \cong \overline{AC}$ and $\overline{AD}$ is 8 cm long. What is the area, in square cm, of $\triangle ABC$?



A. 32 C. $16\sqrt{2}$

B. 64 D. Cannot be determined

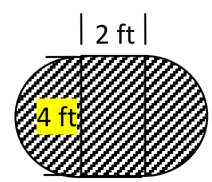
86. If a trapezoid has an area of 72 square meters, one base 10 m long, and another base 8 m long, what is the length, in meters, of its altitude?

A. 4 B. 8 C. 12 D. 16

87. Mr. Reyes has 32 feet of fencing to make a mini-garden area at his backyard. What should the dimensions of the rectangular region be in order to produce the largest possible area for the mini-garden?

A. 6 ft x 10 ft B. 7 ft x 9 ft C. 8 ft x 8 ft D. 7.5 ft x 8.5 ft

88. Find the area of the shaded region.



A. 20.56 ft^2 B. 33.12 ft^2 C. 56.52 ft^2 D. 173.04 ft^2

89. What is the perimeter of an equilateral triangle whose area is $9\sqrt{3}$ cm^2?

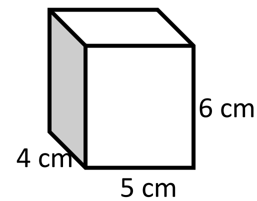
A. 6 cm B. 12 cm C. 18 cm D. 24 cm

90. If the perimeter of a regular hexagon is 24 ft, what is its area?

A. $24\sqrt{3}$ ft^2 B. $12\sqrt{3}$ ft^2 C. $9\sqrt{2}$ ft^2 D. $6\sqrt{2}$ ft^2

CHAPTER 10: Surface Area and Volume

91. The total surface area of the rectangular box shown, is the sum of the areas of the 6 sides. What is the box's total surface area, in square centimeters?



A. 60 C. 120
B. 74 D. 148

92. The volume of a box is 180 cubic in, and the height is 3 in. The length is four times the height. Find the width.

A. 3 in B. 4 in C. 5 in D. 6 in

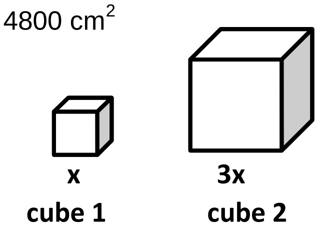
93. Find the volume of a right cylinder with base diameter 10 and height 6.

A. 60π B. 150π C. 300π D. 600π

94. The volume of a cube is 8000 cm^3. What is the total surface area of the cube?

A. 1200 cm^2 B. 2400 cm^2 C. 3600 cm^2 D. 4800 cm^2

95. The ratio of the volume of cube 1 to the volume of cube 2 is ____.

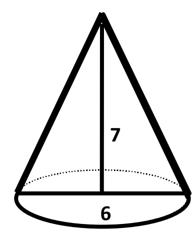


A. 1:3 C. 1:9
B. 1:6 D. 1:27

96. A pyramid has a volume of 64 ft^3 and a height of 8 ft. Find the base area of the pyramid.

A. 8 ft^2 B. 24 ft^2 C. 256 ft^2 D. 512 ft^2

97. Find the volume of the cone (see the figure).



A. 21π C. 84π
B. 42π D. 168π

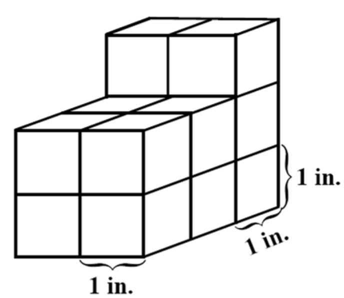
98. Three-fourths of a cylindrical can is filled with water. If the diameter and the volume of the can are 12 cm and 360π cm^3, respectively, find the height of the water.

A. 10 cm B. 7.5 cm C. 5 cm D. 2.5 cm

99. The faces of a triangular pyramid are all equilateral triangles. Find the edge of the pyramid if its total surface area is $100\sqrt{3}$ cm^2.

A. 10 cm B. $10\sqrt{3}$ cm C. 100 cm D. 400 cm

100. One-inch cubes are stacked as shown in the drawing:



What is the total surface area?

A. 19 in.^{2}
B. 29 in.^{2}
C. 32 in.^{2}
D. 38 in.^{2}

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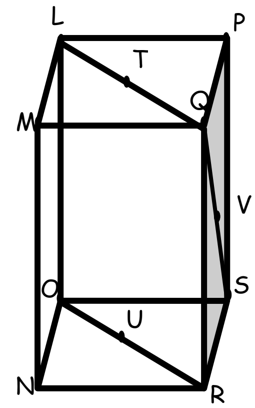
CHAPTER 1: Basic Ideas of Geometry

For Numbers 1 to 4, refer to the figure to the right:



1. Which of the following is a set of collinear points?

A. M, L, P
B. Q, R, S
C. O, U, R
D. S, V, P

> Points that lie on the same line are said to be **collinear**. In the figure, points O, U and R are collinear. Thus, the answer is **C**.

2. V is between points ______.

A. Q and S
B. R and S
C. P and S
D. Q and P

> Given points A, B, and C. B is **between** points A and C if A, B, and C are ***collinear*** and ***AB* + *BC* = *AC***. In the figure, V is between Q and S since the three points are collinear and CV + VS = QS. The answer is **A**.

3. S is the intersection of ______.

A. $\overleftrightarrow{QV}$ and $\overleftrightarrow{PS}$
B. $\overleftrightarrow{RS}$ and $\overleftrightarrow{PS}$
C. $\overleftrightarrow{RS}$ and $\overleftrightarrow{QV}$
D. all of these

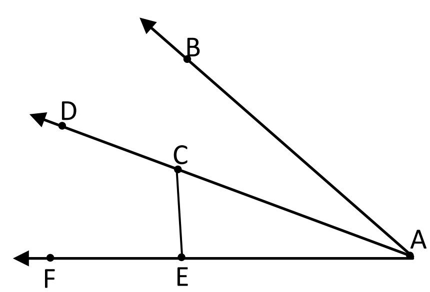
> If two lines intersect, then their intersection is **exactly one point**. Among the choices, it is obvious that both lines **RS** and PS contain the point **S** since their names imply they contain **S**. But take note that line **QV** also contains point **S** (***a line extends infinitely in both directions***). Thus, the intersection of lines **QV** and **PS** as well as that of lines **RS** and **QV** is also S. Therefore, the answer is **D**.

4. Which of the following is a set of coplanar lines?

A. $\overleftrightarrow{LM}, \overleftrightarrow{LP}$ and $\overleftrightarrow{LO}$
B. $\overleftrightarrow{ON}, \overleftrightarrow{OS}$ and $\overleftrightarrow{OR}$
C. $\overleftrightarrow{SQ}, \overleftrightarrow{SO}$ and $\overleftrightarrow{SR}$
D. $\overleftrightarrow{PQ}, \overleftrightarrow{PS}$ and $\overleftrightarrow{PL}$

> Lines that lie on the same plane are said to be coplanar. When lines **LM**, **LP** and **LO** are connected, they form a ***triangular pyramid*** which represents a ***space***, not a plane. Same is true with lines **SQ**, **SO**, and **SR**, as well as with lines **PQ**, **PS**, and **PL**. Lines **ON**, **OS**, and **OR** all lie on the base of the rectangular box, which represents a ***plane***. Thus, these lines are **coplanar**. The answer is **B**.

For Numbers 5 to 7, refer to the figure to the right:



5. E is the midpoint of $\overline{AF}$ if ______.

A. EF = EA
B. EF = ½AF
C. AF = 2·EA.
D. All of these

> If **B** of the ***midpoint*** of segment **AC**, then it divides **AC** into two equal segments **AB** and **BC** (***AB* = *BC***) and each of these segments is half of segment **AC** (***AB* = *BC* = ½*AC***) or **AC** is twice each of the said segments (***AC* = 2*AB* = 2*BC***). If **E** is the midpoint of **AF**, then ***EF* = *EA***, ***EF* = ½*AF***, and ***AF* = 2*EA***. Thus the answer is **D**.

6. Which of the following is true?

A. AC + CE = AE
B. FE + EC = FC
C. m∠ACE + m∠CEA = m∠EAC
D. m∠BAD + m∠DAF = m∠BAF

> If **X**, **Y**, and **Z** are ***collinear*** and **Y** is ***between*** **X** and **Z**, then ***XY* + *YZ* = *XZ***. In the figure, **A**, **C** and **E** are ***not collinear*** so ***AC* + *CE* ≠ *AE***. Also, **F**, **E** and **C** are ***non-collinear points***. Thus ***FE* + *EC* ≠ *FC***. Now, if ray **YW** is in the ***interior*** of angle **XYZ**, then ***m∠XYW* + *m∠YWZ* = *m∠XYZ***. In the figure, angles **ACE**, **CEA** and **EAC** are ***interior angles*** of triangle **ACE**. We ***cannot*** conclude that ***m∠ACE* + *m∠CEA* = *m∠EAC***. On the other hand, ray **AD** is in the ***interior*** of angle **BAF**. Thus, ***m∠BAD* + *m∠DAF* = *m∠BAF***. The answer is **D**.

7. If C is the midpoint of $\overline{AD}$, AD = 4y + 8, and AC = 5y – 8, what is DC?

A. 4 B. 8 C. 12 D. 24

Since **C** is the ***midpoint*** of **AD**, ***2AC = AD***. Substituting the given expressions for **AC**, and **AD**:

$2(5y - 8) = 4y + 8 \rightarrow 10y - 16 = 4y + 8 \rightarrow 10y - 4y = 8 + 16 \rightarrow 6y = 24 \rightarrow y = 4$

And since ***DC = AC***, ***DC = 5y – 8 = 5(4) – 8 = 20 – 8 = 12***. The answer is **C**.

8. What is the inverse of the statement: "If there's a will, then there's a way."?

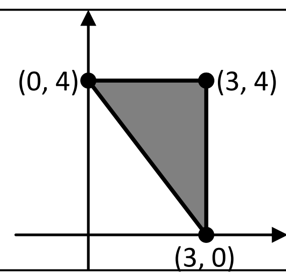
A. If there's no way, then there's no will.
B. If there's no will, then there's no way.
C. If there's a way, then there's a will.
D. If there's a way, then there's no will.

The ***inverse*** of the conditional "***If p then q***" is "***If not p, then not q***". Thus, the ***inverse*** of the statement "***If there's a will, then there's a way***" is "***If there's no will, then there's no way***". The answer is **B**.

9. What type of triangle is the one with vertex coordinates (0, 4), (3, 0) and (3, 4)?

A. scalene B. isosceles C. equilateral D. equiangular

If we make a sketch of the triangle by connecting the vertices, we will form a ***right triangle*** with legs of lengths **3** and **4**. The ***vertex (3, 4)*** is directly above ***(0, 4)*** and to the right of ***(3, 0)***. Thus, the triangle is ***scalene***. The answer is **A**.



10. If a triangle is equilateral, then it is equiangular. △XYZ is not equiangular. What conclusion can you draw?

A. △XYZ is not equilateral.
B. △XYZ is not isosceles.
C. △XYZ is not scalene.
D. △XYZ is not acute.

A ***conditional statement*** (*If p, then q*) is ***logically equivalent*** to its ***contrapositive*** (*If not q, then not p*). The ***contrapositive*** of the statement "***If a triangle is equilateral, then it is equiangular***" is "***If a triangle is not equiangular, then it is not equilateral***". Thus if **ΔXYZ** is ***not equiangular***, then it ***is not equilateral***. The answer is **A**.

GEOMETRY TIP: Geometry is one of the most interesting branches of Mathematics which deals with shapes, size and position of 2-dimensional shapes and 3-dimensional figures. But one look at its varied formulas and it might give jitters to a layman. It can be made quite fascinating if the learner follows some basic rules while studying Geometry.

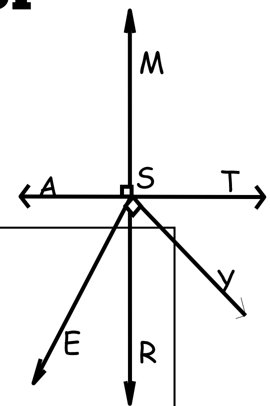
The following tips could be taken into consideration while learning Geometry and its various formulae.

- ***Continuous Practice***: *"Practice Makes a Man Perfect" is an age old adage which is relevant in most cases. More so when one is studying geometry. Learning about geometry, or mathematics as a whole, requires diligence and lots of practice. One can practice learning geometry even outside the boundaries of academic institutions.*

- ***Understand the Definitions***: *The learner should focus on understanding the basic geometrical formulas and its applications.*

- ***Practice by Drawing***: *Pictorial depiction remains in memory for a longer time. So the student of geometry should try and understand the formulae or theorems by drawing them and coloring them.*

CHAPTER 2: Introduction to Proof



For Numbers 11 to 14, refer to the figure to the right:

11. Which of the following are complementary angles?

A. ∠MSA and ∠MST
B. ∠ASE and ∠RSY
C. ∠ESR and ∠RSY
D. ∠ESR and ∠TSY

<u>Definition of Terms:</u>

- **Complementary Angles** – *two angles whose measures have a sum of 90*
- **Supplementary Angles** – *two angles whose measures have a sum of 180*
- **Adjacent Angles** – *two angles that have the same vertex and a common side, but which have no common interior points*
- **Vertical Angles** – *two nonadjacent angles formed by a pair of intersecting lines*

∠MSA and ∠MST form a linear pair, and thus, they are supplementary, *not* complementary (*choice letter A*).

Vertical Angle Theorem *If two angles are vertical angles, then they are congruent.*

In the figure, ∠MAS and ∠TSR are vertical angles. Thus, ∠MAS ≅ ∠TSR. Since ∠MAS is a right angle, ∠TSR is also a right angle or m∠TSR= 90. By Angle Addition Postulate, m∠TSY + m∠RSY = m∠TSR = 90. Thus, ∠TSY and ∠RSY are complementary (*none among the choices*). It is indicated in the figure that ∠YSE is a right angle, thus m∠YSE = 90. By Angle Addition Postulate, m∠ESR + m∠RSY = m∠YSE = 90. Thus, **∠ESR and ∠RSY are complementary** (***choice letter C***). ∠MSA and ∠ASR form a linear pair. Thus, they are supplementary. Since m∠MSA = 90, m∠ASR is also equal to 90. By *Angle Addition Postulate*, m∠ASE + m∠ESR = m∠ASR = 90. Thus, ∠ASE and ∠ESR are complementary (*none among the choices*).

Congruent Complements Theorem

Two angles that are complementary to the same angle (or to congruent angles) are congruent.

Both ∠RSY and ∠ASE are complementary to ∠ESR. Thus, ∠RSY ≅ ∠ASE. The two angles could be complementary if each measures 45. But since the measurements were not indicated in the figure, it cannot be concluded, that ∠RSY and ∠ASE are complementary (*choice letter B*). Both ∠TSY and ∠ESR are complementary to ∠RSY. Thus, ∠TSY ≅ ∠ESR. The two angles could be complementary if each measures 45. But since the measurements were not indicated in the figure, it cannot be concluded, that ∠TSY and ∠ESR are complementary (*choice letter D*).

The answer is **C**.

12. Which of the following are supplementary angles?

A. ∠MSA and ∠ESY
B. ∠MST and ∠ASE
C. ∠ASE and ∠TSY
D. ∠ESR and ∠RSY

∠MSA and ∠ESY are both right angles. Thus, m∠MSA + m∠ESY = 90 + 90 = 180. Therefore, the two angles are supplementary (***choice letter A***). ∠MST is a right angle while ∠ASE is an acute angle. Thus, m∠MST + m∠ASE < 180. Therefore, the two angles are *not* supplementary (*choice letter B*). Both ∠ASE and ∠TSY are acute angles so definitely, the sum of their measures is less than 90 (*choice letter C*). As explained in Item #11, ∠ESR and ∠RSY are complementary, not supplementary (*choice letter D).*

The answer is **A**.

13. Which of the following are congruent angles?

A. ∠RST and ∠ESY
B. ∠RSA and ∠MSA
C. ∠ASE and ∠RSY
D. all of these

∠RST and ∠ESY are both right angles. So they are congruent (***choice letter A***). Also, **∠RSA and ∠MSA** are both right angles. So they are congruent (***choice letter B***). It was established in Item #11 that **∠ASE and ∠RSY** are both complementary to ∠ESR. Thus, ∠ASE ≅ ∠RSY (***choice letter C***).

The answer is **D**.

14. Which of the following are perpendicular?

A. $\overrightarrow{SM}$ & $\overrightarrow{SY}$
B. $\overrightarrow{SA}$ & $\overrightarrow{SR}$
C. $\overrightarrow{SR}$ & $\overrightarrow{SM}$
D. $\overrightarrow{ST}$ & $\overrightarrow{SE}$

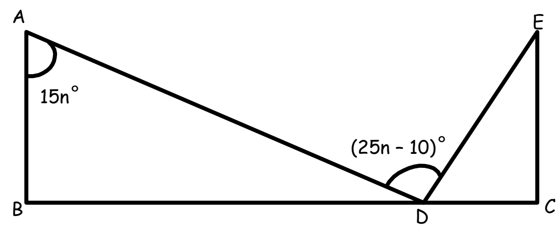
Perpendicular Lines – *two lines that intersect at right angles*

$\overrightarrow{SA}$ **&** $\overrightarrow{SR}$ intersect at right angles, thus they are perpendicular .

The answer is **B**.

For Numbers 15 to 17, refer to the figure to the right:

Given: $\overline{AD} \perp \overline{DE}$



15. m∠ADE = ________.

A. 74
B. 90
C. 99
D. 110

Since $\overline{AD} \perp \overline{DE}$, ∠ADE is a right angle. Thus **m∠ADE = 90**.

The answer is **B**.

16. m∠BAD = ________.

A. 30
B. 45
C. 60
D. 75

m∠ADE = 25n – 10 = 90 ⇒ 25n = 90 + 10 ⇒ 25n = 100 ⇒ ***n = 4*** ⇒ m∠BAD = 15n = 15(4) = **60**.

The answer is **C**.

17. m∠CED = ________.

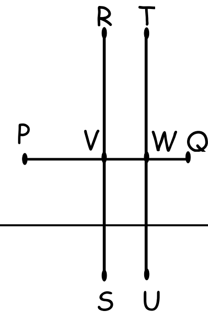
A. 30
B. 45
C. 60
D. 75

The acute angles of a right triangle are complementary. Since m∠BAD = 60, then m∠ADB = 30.

m∠ADB + m∠ADE + m∠EDC = 180
⇒ 30 + 90 + m∠EDC = 180
⇒ m∠EDC = 180 – (30 + 90) = 180 – 120 = 60

Now, ∠EDC and ∠CED are the acute angles of the right triangle CED. Thus, they are complementary and so **m∠CED =30**.

The answer is **A**.



For Numbers 18 to 20, refer to the figure to the right:

18. If $\overline{RS}$ is the perpendicular bisector of $\overline{PQ}$, then ______.

A. RV = VS
B. $\overline{TU} \perp \overline{PQ}$
C. PV = VQ
D. TW = WU

Perpendicular Bisector (of a segment) – *a line, segment, ray or plane that is perpendicular to the segment and bisects it.*

It was not stated that $\overline{PQ}$ is a bisector of $\overline{RS}$. Thus, it cannot be concluded that RV = VS (*choice letter A*). If it was stated that $\overline{TU}$ is parallel to $\overline{RS}$, then it can be concluded that $\overline{TU} \perp \overline{PQ}$ (*choice letter B*). Since $\overline{RS}$ is the perpendicular bisector of $\overline{PQ}$, it divides $\overline{PQ}$ into two congruent segments, PV and VQ. Thus, **PV = VQ** (***choice letter C***).It was not stated that $\overline{PQ}$ is a bisector of $\overline{TU}$. Thus, it cannot be concluded that TW = WU (*choice letter D*).

The answer is **C**.

19. If VW = WQ and $\overline{TU} \perp \overline{PQ}$, then ______.

A. $\overline{PQ}$ is the perpendicular bisector of $\overline{TU}$
B. $\overline{VQ}$ is the perpendicular bisector of $\overline{TU}$
C. $\overline{TU}$ is the perpendicular bisector of $\overline{PQ}$
D. $\overline{TU}$ is the perpendicular bisector of $\overline{VQ}$

Since VW = WQ and $\overline{TU} \perp \overline{PQ}$, **$\overline{TU}$ is the perpendicular bisector of $\overline{VQ}$**.

The answer is **D**.

20. If $\overline{PQ}$ is the perpendicular bisector of $\overline{RS}$, then the ratio of RS to RV is ________.

A. 4:1
B. 3:1
C. 2:1
D. 1:2

If $\overline{PQ}$ is the perpendicular bisector of $\overline{RS}$, RV = VS.

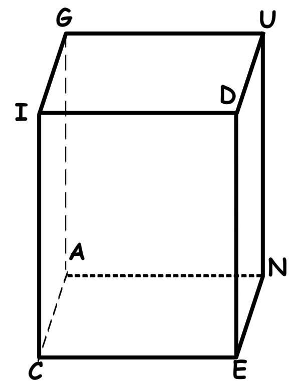
But RV + VS = RS $\Rightarrow$ RV + RV = RS $\Rightarrow$ 2RV = RS $\Rightarrow$ **RS : RV = 2**

The answer is **C**.

GEOMETRY TIP: Ask yourself questions as you move along. ***"Why is this so?"*** and "***Is there any way this can be false?"*** are good questions for every statement, or claim. Back up every statement with a reason! Justify your process.

CHAPTER 3: Parallel Lines and Planes

For Numbers 21 to 24, refer to the rectangular prism to the right:



21. How many sides of the rectangular prism are parallel to $\overline{GA}$?
A. 4 B. 3 C. 2 D. 1

> ***Parallel Lines*** – lines that are coplanar and do not intersect
>
> The lines parallel to line ***GA*** are lines ***UN***, ***IC***, and ***DE***. Thus, there are **three sides** of the rectangular prism are parallel to line ***GA***.
>
> The answer is **B**.

22. How many sides of the rectangular prism are skew to $\overline{GA}$?
A. 7 B. 6 C. 5 D. 4

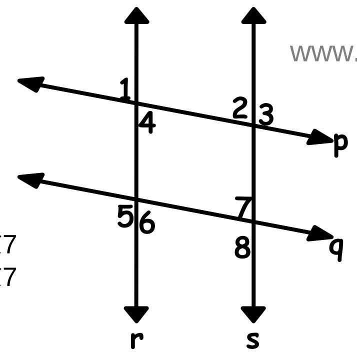
> ***Skew Lines*** **–** lines that are not coplanar and do not intersect
>
> The lines not coplanar with and thus skew to line ***GA*** are lines ***UD***, ***NE***, ***ID***, and ***CE***. Thus, there are **four sides** of the rectangular prism skew to line ***GA***.
>
> The answer is **D**.

23. Which face of the rectangular prism is parallel to IDCE?
A. DUNE C. GACI
B. UGAN D. CANE

> ***Parallel Planes*** – planes that do not intersect
>
> In the figure, we could name ten planes: ***IDCE***, ***UGAN***, ***DUNE***, ***GACI***, ***CANE***, ***IGUD***, ***IDNA***, ***GUEC***, ***GIEN***, and ***UDCA***. Among the planes, all intersect plane ***IDCE*** except **UGAN**.
>
> The answer is **B**.

24. Which line is parallel to $\overleftrightarrow{CN}$?
A. $\overleftrightarrow{IU}$ B. $\overleftrightarrow{GD}$ C. $\overleftrightarrow{GU}$ D. $\overleftrightarrow{ID}$

> Plane ***CANE*** that contains $\overleftrightarrow{CN}$ is parallel to plane ***IGUD***. Among the lines in plane ***IGUD*** (lines ***GU***, ***UD***, ***DI***, ***IG***, ***IU*** and ***GD***), only $\overleftrightarrow{IU}$ appears to be parallel to $\overleftrightarrow{CN}$. The others are skew to $\overleftrightarrow{CN}$.
>
> The answer is **A**.

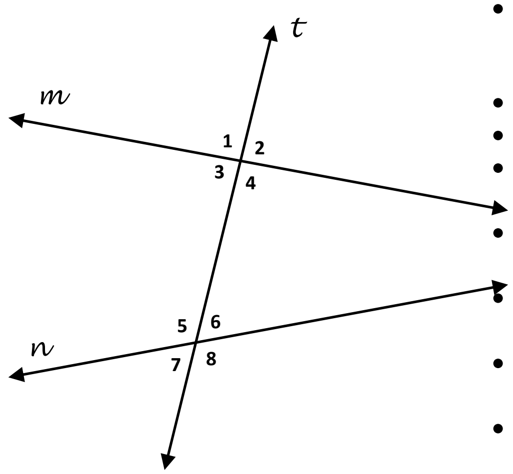


For Numbers 25 to 27, refer to the figure to the right:

25. **If p || q, then ______.**

A. ∠1 ≅ ∠5
B. ∠4 ≅ ∠6
C. ∠3 ≅ ∠7
D. ∠4 ≅ ∠7

<u>Definition of Terms</u>



- **Transversal –** a line that intersects two coplanar lines in two different points (In the figure, transversal ***t*** intersects lines ***m*** and ***n***.)
- **Interior Angles –** angles 3, 4, 5, and 6
- **Exterior Angles –** angles 1, 2, 7, and 8
- **Alternate Interior Angles –** two nonadjacent interior angles on opposite sides of the transversal (∠3 and ∠6; ∠4 and ∠5)
- **Alternate Exterior Angles –** two nonadjacent exterior angles on opposite sides of the transversal (∠1 and ∠8; ∠2 and ∠7)
- **Same-Side Interior Angles –** two interior angles on the same side of the transversal (∠3 and ∠5; ∠4 and ∠6)
- **Same-Side Exterior Angles –** two exterior angles on the same side of the transversal (∠1 and ∠7; ∠2 and ∠8)
- **Corresponding Angles –** two nonadjacent angles on the same side of the transversal such that one is an exterior angle and the other is an interior angle (∠1 and ∠5; ∠3 and ∠7; ∠2 and ∠6; ∠4 and ∠8)

Properties of Parallel Lines Cut by a Transversal

1. Corresponding angles are congruent.
2. Alternate interior angles are congruent.
3. Alternate exterior angles are congruent.
4. Same-side interior angles are supplementary.
5. Same-side exterior angles are supplementary.

In the figure in 3-5, the parallel lines ***p*** and ***q*** are cut by the transversals ***r*** and ***s***. With respect to the transversal ***r***, *∠4 and ∠6* are ***corresponding angles***. Thus, they are congruent (***choice letter B***). ***∠1*** and ***∠5*** are ***same-side exterior angles***, and thus they are ***supplementary***, but not necessarily congruent (*choice letter A*). ***∠3*** and ***∠7*** do not have any relationship, so no conclusion can be made between the two of them (*choice letter* C). ***∠4*** and ***∠7*** are made from different transversals, thus no conclusion can be drawn regarding their measures (*choice letter D*).

The answer is **B**.

<u>GEOMETRY TIP</u>: Whenever possible, draw a diagram. Even though you may be able to visualize the situation mentally, a hand drawn diagram will allow you to label the picture, to add auxiliary lines, and to view the situation from different perspectives.

26. **If $m\angle 6 + m\angle 8 = 180$, then ______.**

A. $r \parallel s$
B. $p \parallel q$
C. $m\angle 6 + m\angle 7 = 180$
D. $m\angle 6 = m\angle 4$

<u>Ways to Prove Two Lines Parallel Using a Transversal</u>

1. Show that a pair of corresponding angles are congruent.
2. Show that a pair of alternate interior angles are congruent.
3. Show that a pair of alternate exterior angles are congruent.
4. Show that a pair of same-side interior angles are supplementary.
5. Show that two coplanar lines are perpendicular to the same line.

Since $m\angle 6 + m\angle 8 = 180$, $\angle 6$ and $\angle 8$ are ***supplementary***. $\angle 6$ and $\angle 8$ are ***same-side interior angles*** with respect to lines ***r*** and ***s*** cut by transversal ***q***. It was shown that a pair of ***same-side interior angles are supplementary***; thus, $r \parallel s$ (***choice letter A***). It cannot be concluded that $p \parallel q$ (*choice letter B*) since none of the five ways to prove them parallel is given. Also, it cannot be concluded that $m\angle 6 + m\angle 7 = 180$ (*choice letter C*) although they are ***congruent*** since they are ***alternate interior angles***. $m\angle 6 = m\angle 4$ only if $p \parallel q$. Since we cannot choose *letter B*, neither we can choose *letter D*.

The answer is **A**.

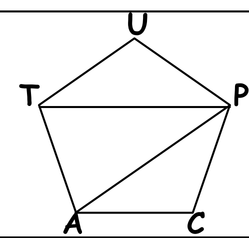
27. **If $r \perp p$, $r \perp q$ and $r \parallel s$, then**

A. $\angle 1 \cong \angle 8$
B. $m\angle 1 + m\angle 8 = 180$
C. Both A and B
D. Neither A nor B

Since $r \perp p$, $r \perp q$ and $r \parallel s$, all the 16 angles formed are ***right angles.*** Thus, **any two angles are congruent and supplementary** at the same time.

The answer is **C**.

28. **Given: $\overline{UT} \cong \overline{UP}$. $\overline{PT} \cong \overline{PA}$. $\angle U \cong \angle C \cong \angle UTA$. $m\angle PAT = 72°$. Find: $m\angle C$.**



A. 72
B. 108
C. 126
D. 144

- *The measure of an interior angle of a regular polygon is* $(n - 2)*180/n$ *where* **n** *is the number of sides .*
- *The sum of the measures of the angles of a triangle is 180.*
- *The base angles of an isosceles triangle are congruent.*

Let assume first that the pentagon is regular, thus the measure of each interior angle is $(5 - 2)*180/5 = 108°$.

Let's verify if it satisfies the given conditions:

$m\angle U = m\angle C = m\angle UTA = 108°$

Since $\overline{UT} \cong \overline{UP}$, $\angle UTP \cong \angle UPT$. Since $\overline{PT} \cong \overline{PA}$, $\angle PTA \cong \angle PAT$.

$m\angle PAT = 72°$, thus $m\angle PTA = 72°$.

$m\angle UTP = m\angle UTA - m\angle PTA = 108° - 72° = 36°$.

$m\angle UPT = m\angle UTP = 36°$

$m\angle U = 180° - (m\angle UPT + m\angle UTP) = 180° - (36° + 36°) = 108°$

The answer is **B**.

29. Find the sum of the exterior angles of a polygon with 18 sides.

A. 360 B. 540 C. 720 D. 900

- *The sum of the measures of the exterior angles of a convex polygon* ***<u>regardless of the number of sides</u>*** *is* ***360****.*

The answer is **A**.

30. If the sum of the measures of five interior angles of a hexagon is 640°, what is the measure of the sixth interior angle?

A. 90 B. 80 C. 70 D. 60

- *The sum of the measures of the angles of a convex polygon of* **n** *sides is* ***(n – 2)180****.*

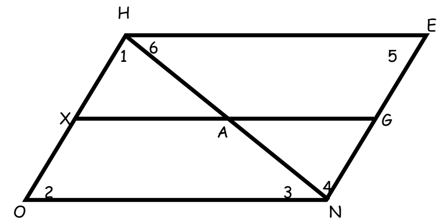
So if a polygon has ***6 sides*** (***hexagon***), the sum of the interior angles will be ***(6 – 2)180 = (4)(180) = 720***. So of the sum of the first five angles is 640, the sixth angle would measure ***720 – 640 = 80***.

The answer is **B**.

<u>GEOMETRY TIP</u>: Review polygons. Do you remember long ago learning that all the angles in a triangle add up to 180? Or that all the angles in a quadrilateral (4-sided shape) add up to 360? Hope so! If you are given a question that involves a figure with many sides, try breaking it up into triangles to make the problem easier.

CHAPTER 4: Congruent Triangles

For Numbers 31 to 34, refer to the figure to the right:

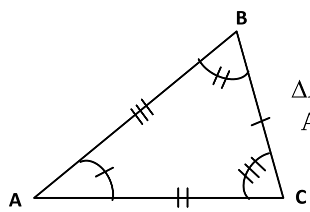


31. If $\triangle OHN \cong \triangle ENH$, then ______.

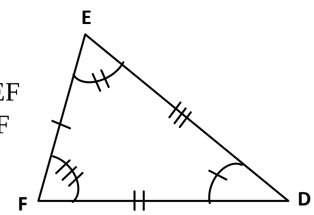
A. $\angle 1 \cong \angle 4$ C. $\angle 3 \cong \angle 5$
B. $\angle 2 \cong \angle 6$ D. $\overline{HX} \cong \overline{NG}$

<u>Definition of Terms:</u>

- **Congruent Figures** – have the same size and shape
- Two triangles are **congruent** *if and only if* there is a **correspondence** between the vertices such that each pair of corresponding sides and each pair of corresponding angles are congruent.



$\Delta ABC \cong \Delta DEF$
$ABC \leftrightarrow DEF$



Corresponding Angles: $\angle A \cong \angle D$ $\angle B \cong \angle E$ $\angle C \cong \angle F$
Corresponding Sides: $\overline{AB} \cong \overline{DE}$ $\overline{BC} \cong \overline{EF}$ $\overline{AC} \cong \overline{DF}$

In the figure in 31, **ΔOHN ≅ ΔENH**. Thus, **OHN ↔ ENH**.

∠1 and ∠4 are corresponding angles of the two triangles (*choice letter A*). ***∠2 and ∠6*** (*choice letter B*) are ***not*** corresponding angles as well as ∠3 and ∠5 (*choice letter C*) . $\overline{HX}$ and $\overline{NG}$ are ***not*** the corresponding sides of the two triangles(*choice letter D*). Thus, by **CPCTC** (**C**orresponding **P**arts of **C**ongruent **T**riangles are **C**ongruent), **∠1 ≅ ∠4** .

The answer is **A**.

32. If $\overline{OH} \cong \overline{HN}$, then ______.

A. $\angle 2 \cong \angle 5$ B. $\overline{OH} \cong \overline{NE}$ C. $\overline{HN} \cong \overline{NE}$ D. $\angle 2 \cong \angle 3$

<u>Definition of Terms:</u>

- In Δ ABC, ∠B is **opposite** $\overline{AC}$ and $\overline{AB}$ is **opposite** ∠C. ∠C is **included** between $\overline{AC}$ and $\overline{BC}$, and $\overline{BC}$ is **included** between ∠C and ∠B.

<u>Isosceles Triangle Theorem</u>
If two sides of a triangle are congruent then the angles opposite those sides are congruent.

In **ΔOHN**, it is given that $\overline{OH} \cong \overline{HN}$. ∠2 is opposite $\overline{HN}$ while ∠3 is opposite $\overline{OH}$. Thus, by ***Isosceles Triangle Theorem***, **∠2 ≅ ∠3** (*choice letter D*). It *cannot* be concluded that **∠2 ≅ ∠5** (*choice letter* A) because it is not given that **ΔOHN ≅ ΔENH**. Same with $\boldsymbol{\overline{OH}}$ ***and*** $\boldsymbol{\overline{NE}}$ (*choice letter B*) $\boldsymbol{\overline{HN} \cong \overline{NE}}$ (*choice letter C*) *cannot* be concluded because it is *not* given that **ΔENH** is isosceles.

The answer is **D**.

33. If △HXA ≅ △NGA, then ______.

A. $\overline{OX} \cong \overline{EG}$ B. $\overline{AN} \cong \overline{XH}$ C. $\overline{ON} \cong \overline{EH}$ D. $\overline{XA} \cong \overline{GA}$

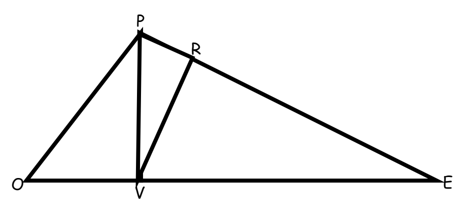
> Since ΔHXA ≅ ΔNGA, HXA ↔ NGA
>
> ⇒ ∠HXA ≅ ∠NGA, ∠XAH ≅ ∠GAN, ∠AHX ≅ ∠ANG
>
> ⇒ $\overline{HX} \cong \overline{NG}$, $\overline{XA} \cong \overline{GA}$, $\overline{HA} \cong \overline{NA}$
>
> Among the six pairs of congruent parts, the one that is in the choices is $\overline{XA} \cong \overline{GA}$ (*choice letter D*). It *cannot* be concluded that $\overline{OX} \cong \overline{EG}$ (*choice letter A*) because each of them is not part of either of the two triangles **ΔHXA and ΔNGA**. Similarly, $\overline{AN}$ ***does not correspond to*** $\overline{XH}$; thus, it is not right to conclude that they are congruent (*choice letter B*). In *choice letter D*, $\overline{ON}$ ***and*** $\overline{EH}$ are corresponding parts of larger triangles **ΔOHN ≅ ΔENH** so no conclusion can be made from them.
>
> The answer is **D**.

34. If A is the midpoint of both $\overline{HN}$ and $\overline{XG}$, then ______.

A. △OHN ≅ △ENH B. △XHA ≅ △GNA C. Both A & B D. Neither A nor B

> **If A is the midpoint of both $\overline{HN}$ and $\overline{XG}$, $\overline{HA} \cong \overline{NA}$** *and* **$\overline{XA} \cong \overline{GA}$.** And by **Vertical Angle Theorem, ∠HAX ≅ ∠GAN. By SAS Postulate, △XHA ≅ △GNA** *(choice letter B).* There is no way to conclude that **△OHN ≅ △ENH** (*choice letter A).*
>
> The answer is **B**.

For Numbers 35 to 37, refer to the figure to the right:



35. If $\overline{PV}$ is the perpendicular bisector of $\overline{OE}$, then ______.

A. V is the midpoint of $\overline{OE}$
B. $\overline{PV}$ is a median of △OPE
C. Both A and B
D. Neither nor B

> **Definition of Terms:**
>
> - A ***median*** of a triangle is a segment from a vertex to the midpoint of the opposite side.
> - An ***altitude*** of a triangle is a segment from a vertex perpendicular to the line containing the opposite side.
> - A ***perpendicular bisector*** of a side of a triangle is a line through the midpoint and perpendicular to the side.
> - *In an isosceles triangle, the altitude of the base bisects the base and bisects the vertex angle.*
> - A point **P** is said to be **equidistant** from two points **A** and **B** if ***PA = PB***.
> - *If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.*
> - *If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.*
> - *If a point is equidistant from the sides of an angle, then it is on the angle bisector.*
> - *If a point is on the angle bisector, then it is equidistant from the sides of the angle*
>
> **If $\overline{PV}$ is the perpendicular bisector of $\overline{OE}$, then**
>
> **V is the midpoint of $\overline{OE}$** (*choice letter A*).
> **$\overline{PV}$ is a median of △OPE** (*choice letter B*).
>
> The answer is **C**.

36. If $\overline{VR} \perp \overline{PE}$, then $\overline{VR}$ is an altitude of both ______.

A. △POV and △PEV
B. △POV and △PRV
C. △PRV and △POE
D. △ERV and △PEV

As defined above, an **altitude** of a triangle is a segment from a vertex perpendicular to the line containing the opposite side. Only triangles with vertex V can have the altitude $\overline{VR}$. Thus, ΔPOE (*in choice letter C*) cannot have $\overline{VR}$ as its altitude. The altitude must be perpendicular to the opposite side. In ΔPOV (*in choice letters A and B*), the opposite side of vertex V is side PO. Thus, $\overline{VR}$ cannot be an altitude. The definition of altitude is satisfied in both triangles **ΔERV and ΔPEV** only (*choice letter D*).

The answer is **D**.

37. If △OPV ≅ △EPV, then ______.

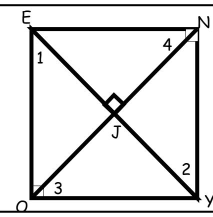
A. △ OPE is isosceles
B. $\overline{PV} \perp \overline{OE}$
C. ∠POV ≅ ∠PEV
D. all of these

If △OPV ≅ △EPV, then

△ OPE is isosceles since $\overline{OP} \cong \overline{EP}$. ***(letter A)***
$\overline{PV} \perp \overline{OE}$ since ∠PVO and ∠PVE are both congruent and supplementary so each measures **90°. *(letter B)***
∠POV ≅ ∠PEV since **CPCTC**. ***(letter C)***

The answer is **D**.

For Numbers 38 to 40, refer to the following figure:



38. If ∠1 ≅ ∠2, then △EOY≅ △YNE by ______.

A. LL Congruence
B. LA Congruence
C. HL Congruence
D. HA Congruence

<u>**Congruence Postulates**</u>

<u>**SSS**</u> **Postulate** *If each of the three sides of one triangle are congruent respectively to corresponding sides of another triangle, then the two triangles are congruent.*

<u>**ASA**</u> **Postulate** *If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.*

<u>**SAS**</u> **Postulate** *If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.*

<u>**AAS Congruence and Right Triangle Congruence**</u>

<u>**AAS**</u> **Congruence** *If two angles and a side opposite one of them in one triangle are congruent to the corresponding parts of another triangle, then the two triangles are congruent.*

<u>**HA Congruence**</u> *If the hypotenuse and one acute angle of a right triangle are congruent to the hypotenuse and one acute angle of another right triangle, then the triangles are congruent.*

<u>**HL**</u> **Congruence** *If the hypotenuse and a leg of one right triangle are congruent to the corresponding parts of a second right triangle, then the triangles are congruent.*

△EOY and **△YNE** are ***right triangles*** sharing a common ***hypotenuse*** $\overline{EY}$. **∠1** and **∠2** are ***corresponding acute angles*** of the two right triangles. Thus, we have an **HA correspondence**.

The answer is **D**.

39. If $\angle 3 \cong \angle 4$ and $\overline{OJ} \cong \overline{NJ}$, then △OJY ≅ △NJE by ______.

A. LL Congruence
B. LA Congruence
C. HL Congruence
D. HA Congruence

> $\angle$**3** and $\angle$**4** are ***corresponding acute angles*** of **△OJY** and **△NJE** while $\overline{OJ}$ and $\overline{NJ}$ ***are corresponding legs*** of the two right triangles. Thus, we have an **LA correspondence**.
>
> The answer is **B**.

40. If $\overline{EN} \parallel \overline{YO}$ and $\overline{EN} \cong \overline{YO}$, then △ONE ≅ △NOY by ______.

A. SSS Congruence
B. SAS Congruence
C. ASA Congruence
D. SSA Congruence

> $\overline{EN} \cong \overline{YO}$ so there's a pair of congruent sides (**S**).
>
> **If $\overline{EN} \parallel \overline{YO}$, then** $\angle$**3** ≅ $\angle$**4 (**alternate interior angles are congruent). So we have a pair of congruent angles (**A**).
>
> $\overline{ON} \cong \overline{NO}$ by Reflexive Property so we have another pair of congruent sides(**S**).
>
> Thus, we have an **SAS correspondence**.
>
> The answer is **B**.

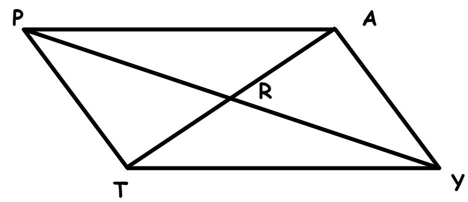
GEOMETRY TIP: In summary, when working with **congruent triangles**, remember to:

1. Mark any given information on your diagram.
2. Look to see if the pieces you need are "parts" of the triangles that can be proven congruent.
3. If not given all needed pieces to prove the triangles congruent, look to see what else you might know about the diagram.
4. Know your definitions! If the given information contains definitions, consider these as "hints" to the solution and be sure to use them.
5. Stay open-minded. There may be more than one way to solve a problem.
6. Look to see if your triangles "share" parts. These common parts are automatically one set of congruent parts.

Remember that proving triangles congruent is like solving a puzzle. Look carefully at the "puzzle" and use all of your geometrical strategies to arrive at an answer.

CHAPTER 5: Using Congruent Triangles and Parallel Lines

For Numbers 41 to 44, refer to the figure to the right:



41. If PAYT is a parallelogram, then ______.
A. PR = YR and TR = AR
B. PA = YT and PT = YA
C. m∠APT = m∠TYA and m∠PAY = m∠YTP
D. All of these

Properties of Parallelograms

- A **parallelogram** is a quadrilateral with two pairs of parallel sides.
- *Opposite sides of a parallelogram are congruent.*
- *Opposite sides of a parallelogram are parallel.*
- *Opposite angles of a parallelogram are congruent.*
- *Consecutive angles of a parallelogram are supplementary.*
- *The diagonals of a parallelogram bisect each other.*

If PAYT is a parallelogram, then :
A. **PR = YR** *and* **TR = AR** (***The diagonals of a parallelogram bisect each other.***)
B. **PA = YT** *and* **PT = YA** (***Opposite sides of a parallelogram are congruent.***)
C. **m∠APT = m∠TYA** *and* **m∠PAY = m∠YTP** (***Opposite angles of a parallelogram are congruent***.)

The answer is **D**.

42. If AT = PY and $\overline{AT} \perp \overline{PY}$, then PAYT is a ______.
A. parallelogram B. rhombus C. square D. rectangle

Proving Quadrilaterals are Parallelograms

- *If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.*
- *If one pair of opposite sides of a quadrilateral are both parallel and congruent, then it is a parallelogram.*
- *If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.*
- *If the consecutive angles of a quadrilateral are supplementary, then the quadrilateral is a parallelogram.*
- *If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.*

Rectangles, Rhombuses, and Squares

- A ***rectangle*** is a quadrilateral with four right angles. The diagonals are congruent.
- A ***rhombus*** is a quadrilateral with four congruent sides. The diagonals are perpendicular.
- A ***square*** is a quadrilateral with four right angles and four congruent sides. It is a rectangle and a rhombus at the same time

Since the diagonals of **PAYT** are ***congruent*** (***rectangle***) and ***perpendicular*** (***rhombus***), then **PAYT** must be a **square**.

The answer is **C**.

43. If PAYT is a rhombus, m∠RPA = 35, then m∠RAP = ______.
A. 145 B. 65 C. 55 D. 35

If **PAYT** is a ***rhombus***, then diagonals ***AT*** and ***PY*** are ***perpendicular***. That makes **ΔPAR** a ***right triangle*** with ***m∠RPA = 90***. The acute angles ***∠RPA*** and ***∠RAP*** are ***complementary***. Thus, ***m∠RAP = 90 - m∠RPA = 90 – 35* = 55**.

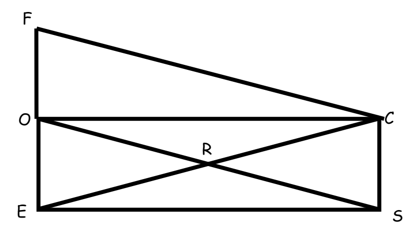
The answer is **C**.

44. If PAYT is a rectangle, then ______.
A. $\overline{AT} \perp \overline{PY}$ B. AT = PY C. m∠PAT = m∠YAT D. AP = RT

If **PAYT** is a ***rectangle*** then diagonals ***AT*** and ***PY*** are ***congruent*** or **AT = PY**.

The answer is **B**.

For Numbers 45 to 47, refer to the figure to the right:



Given: FOSC is a parallelogram and OCSE is a rectangle.

45. △FCE is a/an ______ triangle
A. right C. isosceles
B. scalene D. equilateral

Since **FOSC** is a ***parallelogram***, **FC = OS**.

Since **OCSE** is a ***rectangle***, **OS = EC**.

By ***Transitive Property***, **FC = EC**.

There is no way to prove that third side **FE** of **ΔFCE** is also ***congruent*** to sides **FC** and **EC**. Thus **ΔFCE** is just an **isosceles triangle**.

The answer is **C**.

46. If OE = OC, then ______.
A. △COE is a 45°-45°-90° △ C. both A and B
B. OCSE is a square D. neither A nor B

Since **OCSE** is a ***rectangle***, **ΔCOE** is a right triangle. And since ***OE = CE***, **ΔCOE** is an ***isosceles right triangle*** or a **45°-45°-90° △.**

It follows that **OCSE** is a **square** since it is a ***rectangle*** with ***four congruent sides*** (***OE = OC = SC = SE***).

The answer is **C**.

47. Which of the following pairs of triangles are congruent?
A. △OCR and △SER C. △FOC and △CSE
B. △ORE and △SRC D. All of these

△OCR and △SER are ***congruent*** by **SSS** or **SAS Correspondence**.

△ORE and △SRC are ***congruent*** also by **SSS** or **SAS Correspondence**.

△FOC and △CSE are ***congruent*** by **HL Correspondence**.

The answer is **D**.

48. Which of the following is NEVER true?

A. The legs of a trapezoid are congruent.
B. Opposite angles of a trapezoid are supplementary.
C. Two angles of a trapezoid are right.
D. The bases of a trapezoid are congruent.

<u>**Trapezoids**</u>

- A **trapezoid** is a quadrilateral with exactly one pair of parallel sides.
- The parallel sides of a trapezoid are called **bases**.
- The nonparallel sides are called **legs**.
- The pairs of angles formed by a base and the legs are called **base angles**.
- An **isosceles trapezoid** is a trapezoid with congruent nonparallel sides.
  - *Each pair of base angles of an isosceles trapezoid are congruent.*
  - *The diagonals of an isosceles trapezoid are congruent.*

The legs of a trapezoid can be congruent if the trapezoid is isosceles (*choice letter A*).

Also, in an isosceles trapezoid, opposite angles are supplementary. If the trapezoid is not isosceles, then opposite angles are NOT supplementary (*choice letter B*).

An angle of a trapezoid could be a right angle (*choice letter C*) but it is NOT always the case. Oftentimes, a trapezoid has two acute angles and two obtuse angles.

If the bases of a quadrilateral are congruent and parallel at the same time, then the quadrilateral is ***NOT a trapezoid*** but a ***parallelogram***. **The bases of a trapezoid are NEVER congruent.**

The answer is **D**.

49. Which of the following properties is common to an isosceles trapezoid and a rectangle?

A. The diagonals are congruent.
B. Opposite angles are congruent.
C. Opposite sides are congruent.
D. Consecutive angles are congruent.

Only in rectangles are opposite angles congruent. In trapezoids, opposite angles are NEVER congruent. In isosceles trapezoids, opposite angles are supplementary (*Choice letter B). **Only in rectangles are opposite sides congruent.*** In isosceles trapezoids, only the legs are congruent. In any trapezoid, the bases are NEVER congruent (*Choice letter C).* Since all the angles ***in a rectangle*** are right, ***all the angles are congruent whether they are consecutive angles or opposite angles***. In an isosceles trapezoid, only the base angles are congruent. If the angles belong to different bases, even if they are consecutive, they are NEVER congruent. (*Choice letter D).* The only property common to both an ***isosceles trapezoid*** and a ***rectangle*** is that **the diagonals are congruent**.

The answer is **A**.

50. If at least two midsegments of a triangle are congruent, then the triangle is ______.

A. scalene
B. right
C. isosceles
D. equilateral

The ***midsegment*** of a triangle is formed by joining the midpoints of two sides of the triangle.

<u>**The Midsegment Theorem**</u> *The segment that joins the midpoints of two sides of a triangle is parallel to the third side and has a length equal to half the length of the third side.*

Only in an ***equilateral triangle*** are all the ***three midsegments congruent***. If only ***two midsegments are congruent***, the triangle is just an **isosceles triangle**.

The answer is **C**.

CHAPTER 6: Similarity

51. Which of the following pairs of triangles are similar?

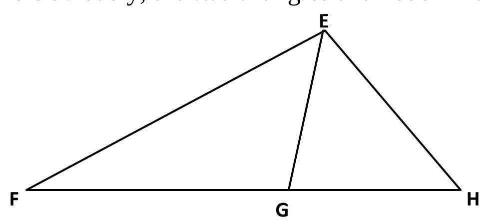
A. Two isosceles right triangles
B. Two triangles with a common base
C. Two overlapping triangles
D. Two scalene triangles

Two polygons are **similar** (~) if their vertices can be matched so that

a. corresponding angles are congruent, and
b. ratios of lengths of corresponding sides are equal.

Using the definition of similar polygons (or triangles), it is easy to disprove the following:

1. ***Two triangles with a common base are similar*** (***choice letter B***): Let ΔABC has sides 3-4-5 with BC = 5 and ΔDBC has sides 6-6-5 with BC = 5. Here, BC is the common base. Since $3 : 6 \neq 4 : 6$, ΔABC is ***not similar*** to ΔDBC.
2. ***Two overlapping triangles are similar*** (***choice letter C***): The figure below shows overlapping triangles (ΔEGH and ΔEFH. Obviously, the two triangles ***are not similar***.



3. ***Two scalene triangles are similar*** (***choice letter D***): A 5-12-13 triangle is ***not similar*** to a 6-7-8 triangle.

Since every ***isosceles right triangle*** is a ***45°-45°-90° triangle***, any two ***isosceles right triangles are similar*** (***choice letter A***).

The answer is **A**.

52. Which of the following pairs of polygons are similar?

A. Two right triangles
B. Two rectangles
C. Two regular pentagons
D. Two kites

For a given pair of polygons to be similar, they must satisfy the definition of similar polygons:
Corresponding angles are congruent, and ratios of lengths of corresponding sides are equal.

Since a ***regular pentagon*** is **ALWAYS equilateral** (*all sides are congruent*) and **ALWAYS equiangular** (*all angles are right*), any **two regular pentagons** are ***ALWAYS similar***.

The answer is **C**.

53. If $\frac{y}{5} = \frac{3}{7}$, then ______.

A. $\frac{y}{5} = \frac{7}{3}$ B. $\frac{5}{y} = \frac{3}{7}$ C. $\frac{y+5}{5} = \frac{8}{7}$ D. 7y = 15

Ratio and Proportion

- For any two numbers, x and y, the **ratio** of x to y is the quotient obtained by dividing x by y.
- A ratio that is expressed in lowest terms is said to be in **simplest form**.
- A **proportion** is an equation stating that two ratios are equal.

$$\frac{a}{b} = \frac{c}{d} \qquad a : b = c : d$$

- In the proportions above, **a** is the first term, **b** is the second term, **c** is the third term, and **d** is the fourth term. The first and fourth terms are called the **extremes**, and the second and third terms are called the **means** of the proportion.

Means-Extremes Products Theorem

- *In a proportion, the product of the means equals the product of the extremes.*

Properties of Proportions

- *If* $\frac{a}{b} = \frac{c}{d}$*, and a, b, c, and d* $\neq 0$*, then each of the following is true.*

a. $\frac{a}{c} = \frac{b}{d}$

b. $\frac{d}{b} = \frac{c}{a}$

c. $\frac{b}{a} = \frac{d}{c}$

d. $\frac{a+b}{b} = \frac{c+d}{d}$

e. $\frac{a-b}{b} = \frac{c-d}{d}$

If y : 5 = 3 : 7, then by **Means-Extremes Products Theorem**, 7y = (5)(3) or **7y = 15**

The other proportions could not be derived from the original one $\left(\frac{y}{5} = \frac{3}{7}\right)$.

The answer is **D**.

54. Leo runs 2 kilometers in 15 minutes. At this rate how long will it take him to run a 25-km marathon?

A. 2 hrs and 22.5 min
B. 2 hrs and 30 min
C. 2 hrs and 45 min
D. 3 hrs and 7.5 min

We can set up the ***ratio and proportion*** as follows:

$$\frac{2\ km}{15\ min} = \frac{25\ km}{x\ min}$$

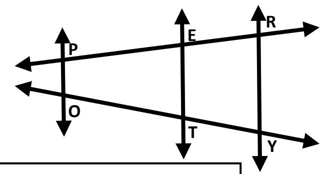
By **Means-Extremes Products Theorem**,

2x = (15)(25) or 2x = 375

x = ***187.5 min*** or **3 hrs and 7.5 min**

The answer is **D**.

55. **In the figure, $\overleftrightarrow{PO} \parallel \overleftrightarrow{ET} \parallel \overleftrightarrow{RY}$, PE = 8, ER = 4 and OT = 10, then TY = ______.**



A. 4 C. 6
B. 5 D. 7

<u>Parallels Proportional Segment Theorem</u>

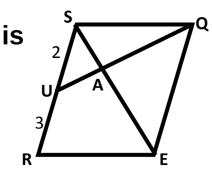
If three parallel lines intersect two transversals, then they divide the transversals proportionally.

Thus, in the figure for item ***6-5***, **PE : OT = ER : TY** ⇒ **8 : 10 = 4 : TY**

Solving for ***TY***, **TY = (10)(4)/8** or **TY = 5**.

The answer is **B**.

56. **SQER is a parallelogram and U is a point on the side SR such that SU = 2 and UR = 3. A is the point of intersection of SE and UQ. The ratio of SA:AE is equal to ______.**



A. 2:3 B. 1:2 C. 1:3 D. 2:5

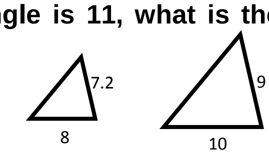
By ***AA Similarity***, **ΔUSA ~ ΔQEA** since **∠SAU ≅ ∠EAQ** (*vertical angles*) and **∠USA ≅ ∠QEA** (*alternate interior angles*: *side **SR** is parallel to side QE*).

Thus, SA : AE = SU : QE. But QE = SR = SU + UR = 2 + 3 = 5. ⇒ **SA : AE = 2 : 5**

The answer is **D**.

57. **The two triangles are similar. If the length of the missing side of the larger triangle is 11, what is the length of the missing side of the smaller triangle?**

A. 13.75 B. 10.2 C. 8.8 D. 7.6



Let ***x*** be the missing side of the smaller triangle:

x : 11 = 8 : 10 → ***10x = (8)(11) = 88*** → **x = 8.8**

The answer is **C**.

58. **What must be the value of y in order that $\overline{OW}$ be parallel to $\overline{ER}$?**

A. 8 C. 12
B. 10 D. 15

If $\overline{OW}$ would be parallel to $\overline{ER}$, then

y : 5 = 6 : 3 → ***3y = (5)(6) = 30*** → **y= 10**

The answer is **B**.

59. If the sides of a quadrilateral are 8, 14, 12, and 20 cm and the longest side of a similar quadrilateral is 30 cm, how long is the shortest side of this quadrilateral?

A. 6 cm B. 10 cm C. 12 cm D. 14 cm

Since the longest side of the first quadrilateral is **20 cm**, the ratio of the sides of the two similar quadrilaterals is ***20:30***. The shortest side of the first quadrilateral is **8 cm**. Let x be the shortest side of the second quadrilateral.

Thus, $8 : x = 20 : 30 \rightarrow 20x = (30)(8) \rightarrow 20x = 240 \rightarrow \mathbf{x = 12}$

The answer is **C**.

60. What is the geometric mean between 18 and 72?

A. 36 B. $24\sqrt{2}$ C. $48\sqrt{2}$ D. 45

Geometric Mean. The geometric mean of a set of positive data is defined as the ***nth root of the product of all the members of the set***, where ***n*** is the number of members.

Given the set of positive numbers $x_1, x_2, x_3, \ldots, x_n$, the ***geometric mean*** is: $\sqrt[n]{x_1 * x_2 * x_3 * \ldots * x_n}$

Examples:

1. The geometric mean of 2, 4 and 8 is $\sqrt[3]{2*4*8} = \sqrt[3]{64} = \mathbf{4}$.
2. The geometric mean between 3 and 27 is $\sqrt{3*27} = \sqrt{81} = \mathbf{9}$.

Thus, the geometric mean between 18 and 72 is $\sqrt{18*72}$. Since the product of 18and 72 is a large number, it should be noticed that 72 is (18)(4). So $\sqrt{18*72} = \sqrt{18*18*4} = 18*2 = \mathbf{36}$.

The answer is **A**.

GEOMETRY TIP: The little things about **ratios and proportions**…

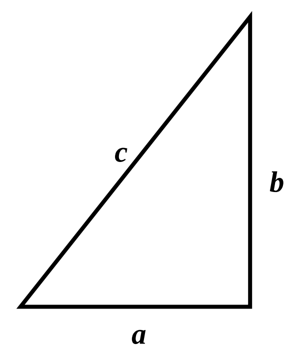
- *Compare values with the same units*
- *Keep track of all given units for a proportion problem*
- *Check your proportion problems with cross multiplication, if possible.*
- *Never try to work with decimals in ratios or proportions. Convert decimals into whole numbers by multiplying 10, 100, 1000, etc. to all terms.*

CHAPTER 7: Right Triangles

61. Which of the following is NOT true about right triangles?

A. The longest side is the longer leg.
B. The hypotenuse is opposite 90°.
C. The legs are perpendicular.
D. The acute angles are complementary.

A ***right triangle*** is a triangle in which ***one angle measures 90°***. It is a geometric figure with three sides, where ***the two shorter sides meet at a right angle (choice letter C)***. The ***acute angles*** of a right triangle are always ***complementary (choice letter D)***. The ***Pythagorean Theorem*** named after the Greek mathematician ***Pythagoras*** who developed a formula to find the lengths of the sides of any right triangle. The theorem can be written as an equation: $a^2 + b^2 = c^2$



The longest side of a right triangle "***c***", is called ***Hypotenuse*** (***choice letter A is wrong)***and the other ***two shorter sides*** "***a***" and "***b***" are referred as ***legs*** of the triangle. The ***side opposite the right angle*** is always the ***Hypotenuse (choice letter B)***.

The answer is **A**.

62. If the sum of the squares of the lengths of the two shorter sides of a triangle is less than the square of the length of the longest side, the triangle is ______.

A. acute B. obtuse C. scalene D. right

If $a < b < c$ are lengths of the sides of a triangle and

a. $a^2 + b^2 < c^2$***, then the triangle is an obtuse triangle.***
b. $a^2 + b^2 > c^2$***, then the triangle is an acute triangle.***

Thus, if the sum of the squares of the lengths of the two shorter sides of a triangle is ***less than*** the square of the length of the longest side ($a^2 + b^2 < c^2$), the triangle is **obtuse**.

The answer is **B**.

63. Which of the following sets of numbers forms a Pythagorean Triple?

A. {1, 2, 3}
B. {20, 21, 29}
C. {9, 16, 25}
D. $\{\sqrt{3}, \sqrt{4}, \sqrt{7}\}$

A **Pythagorean Triple** is any three whole numbers ***a***, ***b***, and ***c*** that satisfy the equation $a^2 + b^2 = c^2$.
Since in choice D ($\{\sqrt{3}, \sqrt{4}, \sqrt{7}\}$), only $\sqrt{4}$ is a whole number, the set could ***not*** be a ***Pythagorean Triple*** (even if it satisfies the equation $a^2 + b^2 = c^2$).

Choices ***A*** and ***C*** do not satisfy the equation $a^2 + b^2 = c^2$:

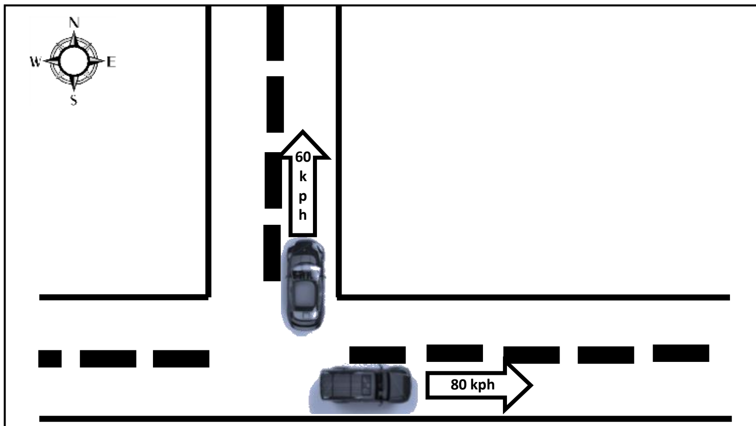
A. $1^2 + 2^2 \neq 3^2$
C. $9^2 + 16^2 \neq 25^2$

Only {***20, 21, 29***} satisfies the definition of a ***Pythagorean Triple*** .

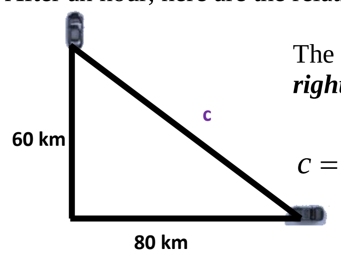
The answer is **B**.

64. Two cars start at the same time and in the same place. One travels due north at 60 kph while the other due east at 80 kph. How far are they from each other after one hour?

A. 140 km
B. 100 km
C. 70 km
D. 50 km



After an hour, here are the relative positions of the two cars:



The ***distance c***, is the ***hypotenuse*** of the ***right triangle*** formed.

$$c = \sqrt{60^2 + 80^2} = \sqrt{3600 + 6400}$$

$$c = \sqrt{10000} = 100\ km$$

The answer is **B**.

65. Which of the following are the sides of an acute triangle?

A. 6, 8, 9
B. 15, 20, 27
C. 21, 28, 35
D. 27, 36, 48

If $a < b < c$ are lengths of the sides of a triangle and $a^2 + b^2 > c^2$, then the triangle is an ***acute triangle.***

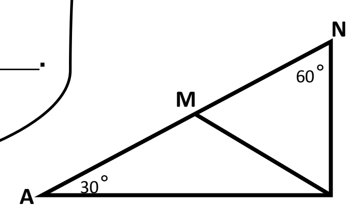
A ***3-4-5 triangle*** is a ***right triangle***. Any triangle whose sides are proportional to the ***3-4-5 triangle*** is also a ***right triangle***. This fact can be used to determine which among the choices is an ***acute triangle***.

A. **6, 8, 9** $\Rightarrow$ **(3, 4, 5)x2 = (6, 8, 10)** $\Rightarrow$ $6^2 + 8^2 = 10^2 > 9^2$ $\Rightarrow$ **the Δ is acute**
B. **15, 20, 27** $\Rightarrow$ **(3, 4, 5)x5 = (15, 20, 25)** $\Rightarrow$ $25^2 < 27^2$ $\Rightarrow$ ***the Δ is obtuse***
C. **21, 28, 35** $\Rightarrow$ **(3, 4, 5)x7 = (21, 28, 35)** $\Rightarrow$ $35^2 = 35^2$ $\Rightarrow$ ***the Δ is right***
D. **27, 36, 48** $\Rightarrow$ **(3, 4, 5)x9 = (27, 36, 45)** $\Rightarrow$ $45^2 < 48^2$ $\Rightarrow$ ***the Δ is obtuse***

The answer is **A**.

66. Given △ANE. If $\overline{EM}$ is an altitude, then △MEN is ______.

A. an equilateral triangle
B. an isosceles triangle
C. a right triangle
D. an obtuse triangle



If $\overline{\mathbf{EM}}$ is an ***altitude***,

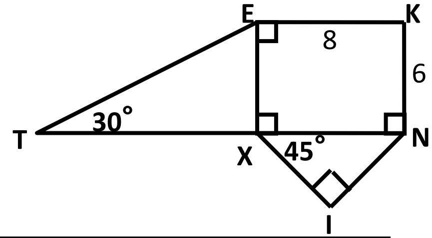
then $\overline{EM} \perp \overline{MN} \rightarrow$ ***∠NME is a right angle***

thus, **△MEN is a right triangle**.

The answer is **C**.

GEOMETRY TIP: Review **right triangles** for the UPCAT. A right triangle is one that has an angle of 90 degrees at one of its corners. Every right triangle comes from cutting a rectangle in half along the diagonal. A square is a type of rectangle, and when you cut a square into two triangles, these are called ***"Isosceles Right Triangles"*** They appear often on the UPCAT! The two shorter sides are called "legs" and the longest side is called the "hypotenuse". If the legs have lengths a and b and the hypotenuse has length c, then $a^2 + b^2 = c^2$. This is called the ***"Pythagorean Theorem"*** and you will need to know it.

67. Find the area of hexagon *TEKNIX*.



A. $26 + 6\sqrt{3} + 8\sqrt{2}$

B. 73

C. $64 + 18\sqrt{3}$

D. $26 + 14\sqrt{6}$

There are three special figures in hexagon ***TEKNIX***:

1. **Rectangle *EKNX*** (*A rectangle is a quadrilateral with four right angles.*) – Since quadrilateral ***EKNX*** has already 3 right angles, the fourth angle must also be a right angle (***360° - 3*90° = 360° - 270° = 90°***). Also, a rectangle has all properties of a parallelogram like "opposite sides of a parallelogram are congruent." Thus, ***EK =XN = 8*** and ***EX = KN = 6***.

2. ***ΔNIX*** is an ***isosceles right Δ*** (or a ***45°-45°-90° Δ***) since it is given that ***∠NXI*** is ***45°*** and ***∠XNI*** is ***90°*** (thus, ***∠XIN = 180° - (45° + 90°) = 45°***). In an ***isosceles right triangle***, each leg is $\frac{\sqrt{2}}{2}$ times the hypotenuse.

   Thus, **XI = NI** = $\left(\frac{\sqrt{2}}{2}\right)(XN) = \left(\frac{\sqrt{2}}{2}\right)(8) = 4\sqrt{2}$

3. ***ΔETX*** is a ***30°-60°-90° Δ*** since it is given that ***∠ETX*** is ***30°***. Also, since ***∠EXN= 90°*** and it forms a ***linear pair*** with ***∠ETX***, it can be concluded that ***∠ETX = 90°.*** Thus, ***∠TWX = 180° - (30° + 90°) = 60°***. In a ***30°-60°-90° Δ,*** the ***longer leg*** is $\sqrt{3}$ ***times*** the ***shorter leg*** and the ***hypotenuse*** is ***twice the shorter leg***. Also, in a ***30°-60°-90° Δ,*** the ***shorter leg*** is ***opposite the 30° angle*** and the ***longer leg is opposite the 60° angle***. Thus, ***XT*** = $(\sqrt{3})(EX) = (\sqrt{3})(6) = 6\sqrt{3}$ and ***ET = 2(EX) = 2(6) = 12***.

The area of hexagon *TEKNIX* is the sum of the areas of the three special figures above. Thus,

1. ***Area of Rectangle EKNX = (EK)(KN) = (8)(6) = 48***
2. ***Area of ΔNIX = ½(XI)(NI) = ½(*** $4\sqrt{2}$ ***)(*** $4\sqrt{2}$ ***) = 16***
3. ***Area of ΔETX= ½(XT)(EX) = ½(*** $6\sqrt{3}$ ***)(6) =*** $18\sqrt{3}$

Area of hexagon *TEKNIX* = ***48 + 16 +*** $18\sqrt{3}$ **= 64 +** $18\sqrt{3}$

The answer is **C**.

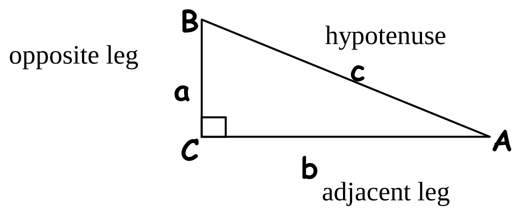
<u>GEOMETRY TIP</u>: **<u>Solving Special Right Triangles</u>**. When solving special right triangles, remember that a *30-60-90 triangle* has a hypotenuse twice as long as one of the sides, and a *45-45-90 triangle* has two equal sides.

68. In a right triangle, as an acute angle gets smaller, ______________________.

A. the cosine gets closer to 1 and the sine gets closer to 0
B. the sine gets closer to 1 and the cosine gets closer to 0
C. the cosine gets closer to 0 and the tangent gets closer to 1
D. the tangent gets closer to 0 and the sine gets closer to 1

The Tangent, Sine and Cosine Ratios

- A **trigonometric ratio** is a ratio of the lengths of two sides of a right triangle.



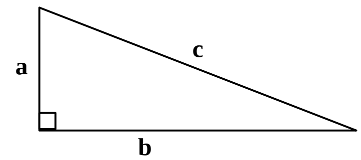
- The three ratios are given below for ∠A. Note that the tangent of A is usually abbreviated **tan A**, the sine of A is usually abbreviated **sin A**, and the cosine of A is usually abbreviated **cos A**.
- **SOH-CAH-TOA:**
  - sin A = $\frac{\text{length of the opposite leg}}{\text{length of the hypotenuse}} = \frac{a}{c}$
  - cos A = $\frac{\text{length of the adjacent leg}}{\text{length of the hypotenuse}} = \frac{b}{c}$
  - tan A = $\frac{\text{length of the opposite leg}}{\text{length of the adjacent leg}} = \frac{a}{b}$
- In ΔABC, a, b, and c are always positive and c is always greater than a or b. Thus, the ratio $\frac{a}{c}$ or $\frac{b}{c}$ ranges between 0 and 1. As angle A gets smaller, angle B gets larger and consequently, side a gets shorter and side b gets longer. Thus, as angle A gets smaller, $\frac{a}{c}$ becomes smaller and gets closer to 0 and $\frac{b}{c}$ becomes larger and gets closer to 1. The conclusion is the same as in *choice letter A*: **the cosine gets closer to 1 and the sine gets closer to 0**. What happens to $\frac{a}{b}$? Since a gets shorter and b gets longer, $\frac{a}{b}$ or tan A gets smaller and it approaches 0.

- Also, tan A = $\frac{\sin A}{\cos B}$. Thus, as angle A gets smaller, sin A gets closer to 0 and cos A gets closer to 1, tan A = $\frac{\sin A}{\cos B} \approx \frac{0}{1} \approx 0$.

The answer is **A**.

69. In the figure the right triangle has side lengths a, b, and c, where c is the length of the hypotenuse. a^2 can be found by

A. adding b^2 and c^2.
B. squaring the sum of b and c
C. Both A and B
D. Neither A nor B



Since the Δ above is a right Δ, ***Pythagorean Theorem*** applies: $a^2 + b^2 = c^2$

Solving for a^2, $a^2 = c^2 - b^2$ (subtract a^2 from each side of the formula). This is not the same as ***Choice letter A*** (*adding b^2 and c^2*). Neither it is the *square of the sum of b and* c (***Choice letter B***)

The answer is **D**.

70. Which of the following is equal to sin θ?

A. (tan θ)(cos θ) B. tan θ/cos θ C. cos θ/tan θ D. 1/cos θ

In ΔABC,

- $\sin\theta = \dfrac{a}{c}$
- $\cos\theta = \dfrac{b}{c}$
- $\tan\theta = \dfrac{a}{b}$

A. $(\tan\theta)(\cos\theta) = \left(\dfrac{a}{b}\right)\left(\dfrac{b}{c}\right) = \dfrac{a}{c} = \sin\theta$

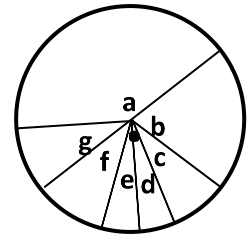
B. $\tan\theta/\cos\theta = \dfrac{a}{b}\Big/\dfrac{b}{c} = \left(\dfrac{a}{b}\right)\left(\dfrac{c}{b}\right) = \dfrac{ac}{b^2}$

C. $\cos\theta/\tan\theta = \dfrac{b}{c}\Big/\dfrac{a}{b} = \left(\dfrac{b}{c}\right)\left(\dfrac{b}{a}\right) = \dfrac{b^2}{ac}$

D. $1/\cos\theta = 1\Big/\dfrac{b}{c} = \dfrac{c}{b}$

The answer is **A**.

CHAPTER 8: Circles



71. If a + b = 200°, and c + d + e + f = 140°, what is the number of degrees in angle g?

A. 10° C. 30°

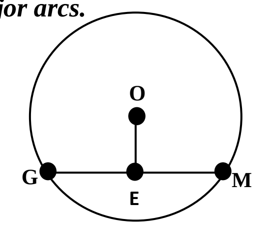
B. 20° D. 45°

> ***The sum of the labeled central angles of the circle is 360°:***
>
> ***a + b + c + d + e + f + g = 360 → 200 + 140 + g = 360 → 340 + g = 360 →*** **g = 20**
>
> The answer is **B**.

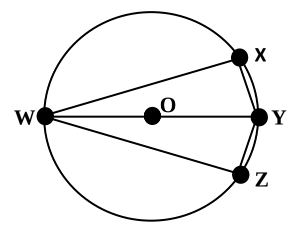
72. Given the circle O in the figure. If $\overline{OE} \perp \overline{GM}$, then

A. $\overline{EO} \cong \overline{GM}$ C. $\overline{EO} \cong \overline{OM}$

B. $\overline{EO} \cong \overline{OG}$ D. $\overline{GE} \cong \overline{EM}$

> ***If a diameter is perpendicular to a chord, then it bisects the chord and its minor and major arcs.***
>
> 
>
>
> In the figure, $\overline{OE}$ contains the center (O) of the circle, thus it lies on a diameter of the circle. Since $\overline{OE} \perp \overline{GM}$, $\overline{OE}$ bisects chord $\overline{GM}$. Thus, $\overline{GE} \cong \overline{EM}$.
>
> The answer is **D**.

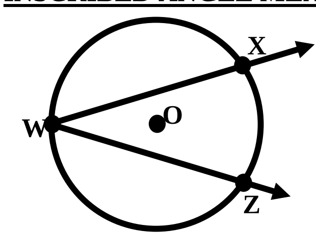
For Numbers 73 and 74, refer to the figure to the right:



73. If measure of arc XZ = 100, then

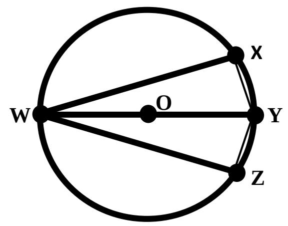
A. m∠XWZ = 100 C. m∠XWY = 50

B. m∠XWZ = 50 D. m∠ZWY = 50

> An ***inscribed angle*** is an angle with vertex on a circle and sides that contain chords of the circle. In the figure, ***∠XWZ*** is an ***inscribed angle*** and ***arc XZ*** is its **intercepted arc**.
>
> **<u>INSCRIBED ANGLE MEASURE</u>**
>
> 
>
>
> ***The measure of an inscribed angle is half the measure of its intercepted arc.***
>
> Thus, if measure of arc XZ is 100, then **m∠XWZ = ½(100) = 50**.
>
> The answer is **B**.

74. Which of the following is ALWAYS true?

A. m∠XWZ = 90 C. m∠XWY = 90
B. m∠WXY = 90 D. m∠XYW = 90



An angle inscribed in a semicircle is a right angle. Also, if an inscribed angle is a right angle, its intercepted arc is a semicircle.

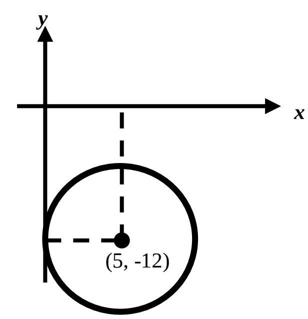
In the figure, ∠WXY and ∠WZY are each inscribed in a semicircle. Thus, m∠WZY = m∠WXY = 90.

The answer is **B**.

75. A circle with center (5, -12) is tangent to the *y*-axis in the standard (*x*, *y*) coordinate plane. What is the radius of this circle?

A. 13 B. 12 C. 7 D. 5

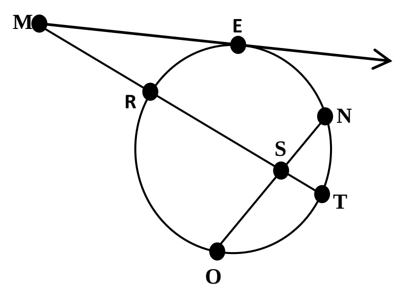
If we draw a quick sketch of the circle:



Since the circle is ***tangent to the y-axis***, its center is ***horizontally* 5 units *away from the y-axis***. That horizontal distance is also the ***radius of the circle***.

The answer is **D**.

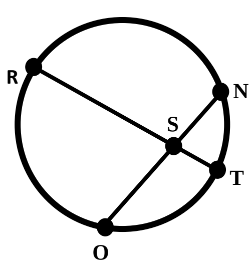
For Numbers 76 and 77, refer to the figure to the right:



76. If measure of arc RN = 110 and measure of arc OT = 60, then

A. m∠RSN = 170 C. m∠TSO = 50
B. m∠RSN = 85 D. m∠TSO = 25

The measure of an angle formed by two chords intersecting inside a circle is one half the sum of the intercepted arcs.



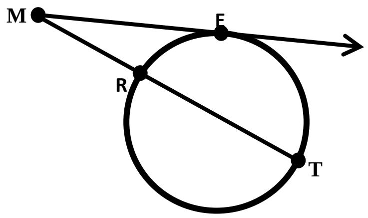
Thus, m∠RSN = ½(m $\overset{\frown}{RN}$ + m $\overset{\frown}{OT}$)
= ½(110 + 60)
= ½(170) = 85

The answer is **B**.

77. If measure of arc ET = 100 and measure of arc RE = 50, then

A. m∠EMR = 150 C. m∠EMR = 50
B. m∠EMR = 75 D. m∠EMR = 25

The measure of an angle formed by a secant and a tangent drawn from a point in the exterior of a circle is equal to half the difference of the measures of the intercepted arcs.

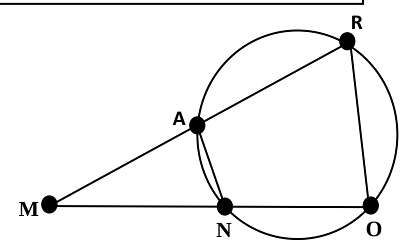


Thus,

$m\angle EMR = \frac{1}{2}(m\overset{\frown}{ET} - m\overset{\frown}{RE}) = \frac{1}{2}(100 - 50) = \frac{1}{2}(50) = \mathbf{25}$

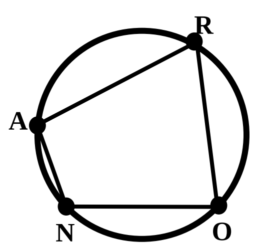
The answer is **D**.

For Numbers 78 to 80, refer to the figure to the right:



78. Which angles of quadrilateral ARON are supplementary?

A. ∠RON and ∠RAN C. Both A and B
B. ∠ANO and ∠ARO D. Neither A nor B



If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

In quadrilateral ***ARON***, there are ***two pairs of opposite angles***:

∠RON and ∠RAN
and
∠ANO and ∠ARO

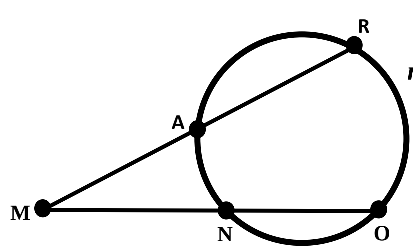
The answer is **C**.

79. If measure of arc RO = 90 and measure of arc AN = 30, then m∠RMO = _______.

A. 10 B. 20 C. 30 D. 40

The measure of an angle formed by two secants drawn from a point in the exterior of a circle is equal to half the difference of the measures of the intercepted arcs.

Thus,



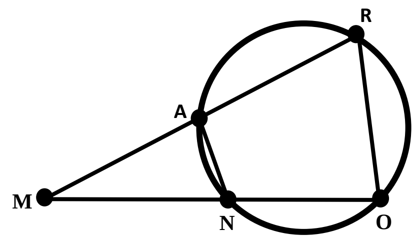
$m\angle RMO = \frac{1}{2}(m\overset{\frown}{RO} - m\overset{\frown}{AN})$
$= \frac{1}{2}(90 - 30)$
$= \frac{1}{2}(60) = \mathbf{30}$

The answer is **C**.

80. If $\overline{NA} \perp \overline{MR}$, then ___________.

A. m∠MAN > m∠RAN
B. m∠MAN = m∠RON
C. m∠MAN = m∠ARO
D. m∠ARO = m∠ANO

If $\overline{NA} \perp \overline{MR}$, then ***∠MAN and ∠RAN are right angles***. So ***m∠MAN = m∠RAN = 90***. In #78, it was concluded that ∠RAN and ∠RON are supplementary because they are opposite angles of quadrilateral ARON inscribed in the circle. Thus, ***m∠RON = 180 – 90 = 90***. ∠RON is also a right angle. Since all right angles are congruent, **m∠MAN= m∠RON**.

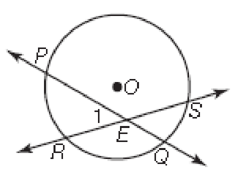


The answer is **B**.

GEOMETRY TIP:

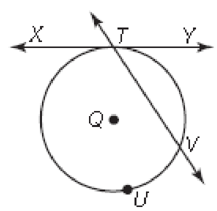
Intersections On or Inside a Circle A line that intersects a circle in exactly two points is called a **secant**. The measures of angles formed by secants and tangents are related to intercepted arcs.

- If two secants or chords intersect in the interior of a circle, then the measure of the angle formed is one half the sum of the measure of the arcs intercepted by the angle and its vertical angle.



$m\angle 1 = \frac{1}{2}(m\widehat{PR} + m\widehat{QS})$

- If a secant (or chord) and a tangent intersect at the point of tangency, then the measure of each angle formed is one half the measure of its intercepted arc.



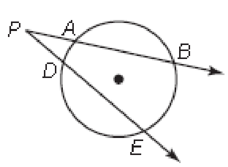
$m\angle XTV = \frac{1}{2} m\widehat{TUV}$

$m\angle YTV = \frac{1}{2} m\widehat{TV}$

Secants, Tangents, and Angle Measures

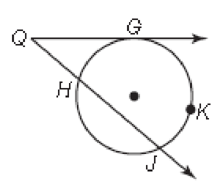
Intersections Outside a Circle If secants and tangents intersect outside a circle, they form an angle whose measure is related to the intercepted arcs.

If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is one half the difference of the measures of the intercepted arcs.



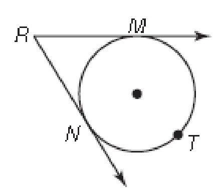
$\overrightarrow{PB}$ and $\overrightarrow{PE}$ are secants.

$m\angle P = \frac{1}{2}(m\widehat{BE} - m\widehat{AD})$



$\overrightarrow{QG}$ is a tangent. $\overrightarrow{QJ}$ is a secant.

$m\angle Q = \frac{1}{2}(m\widehat{GKJ} - m\widehat{GH})$



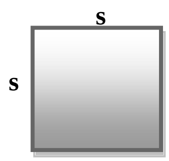
$\overrightarrow{RM}$ and $\overrightarrow{RN}$ are tangents.

$m\angle R = \frac{1}{2}(m\widehat{MTN} - m\widehat{MNC})$

CHAPTER 9: Area and Perimeter of Polygons

81. The area of a square is 225 sq cm. How long is one side of the square?

A. 15 cm B. 20 cm C. 25 cm D. 56.25 cm



Area of square with side ***s***: **A = s²**

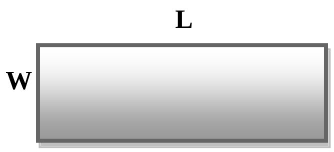
Side of the square with area ***A***: $s = \sqrt{A}$

Thus, if the ***area = 225 cm²***, $s = \sqrt{225} = 15\ cm$

The answer is **A**.

82. A rectangle is thrice as long as it is wide. If the length of the rectangle is 9 inches, what is the rectangle's perimeter, in inches?

A. 12 B. 24 C. 36 D. 72



Perimeter of rectangle with length ***L*** and width ***W***: **P = 2(L + W)**

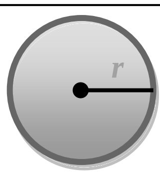
It is given that ***L = 9 in***, and it is thrice the width. Thus, ***W = 3 in***.

P = 2(9 + 3) = 2(12) = 24 in

The answer is **B**.

83. A circular chip has a radius of $\frac{5}{6}$ cm. When lying flat, how much area does the coin cover, in square cm?

A. 5π/6 B. 5π/3 C. 25π/9 D. 25π/36

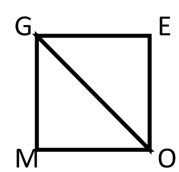


Area of circle with radius ***r***: $A = \pi r^2$

Given that $= \frac{5}{6} cm$ **,** $A = \pi\left(\frac{5}{6}\right)^2 = \frac{25\pi}{36}$

The answer is **D**.

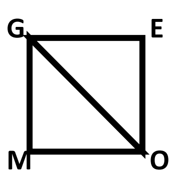
84. Square *GEOM* has a perimeter of 52 inches. How many inches long is diagonal $\overline{GO}$?



A. 26 C. $13\sqrt{3}$

B. $13\sqrt{2}$ D. 13

The ***diagonal d*** of a square with ***side s*** is:



$d = s\sqrt{2}$

The ***perimeter*** of a square with ***side s*** is:

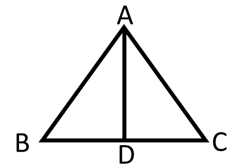
$P = 4s$

Thus, the ***side s*** of square ***GEOM*** is: $s = \frac{P}{4} = \frac{52}{4} = 13\ in$.

and the ***diagonal GO*** is: $d = s\sqrt{2} = 13\sqrt{2}\ in$

The answer is **B**.

85. **In the figure, $\overline{AB} \cong \overline{AC}$ and $\overline{AD}$ is 8 cm long. What is the area, in square cm, of $\triangle ABC$?**



A. 32
B. 64
C. $16\sqrt{2}$
D. Cannot be determined

There are ***3 ways*** to solve for the ***area of a triangle***:

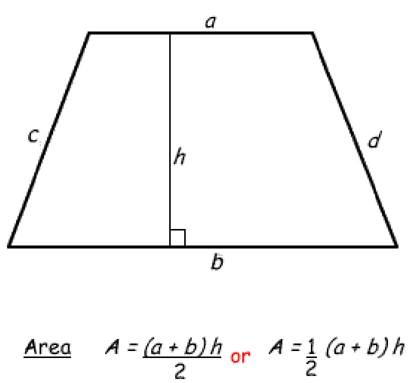
1. The most commonly-used formula is $\boldsymbol{A = \frac{bh}{2}}$. If we know the ***base b*** and the ***height*** (or ***altitude***) ***h*** of the triangle, then the formula can be used. in the figure, only the ***altitude AD*** (***8 cm***) is given. The ***base BC*** is unknown so we ***cannot*** use the formula.
2. Another formula in finding the area of a triangle is ***the Heron's Formula***: $\boldsymbol{A = \sqrt{s(s-a)(s-b)(s-c)}}$ where ***a***, ***b***, and ***c*** are the ***sides*** of the triangle, and ***s*** is the ***semi-perimeter***: $\boldsymbol{s = \frac{a+b+c}{2}}$. Since no side of the triangle in the figure is given, we ***cannot*** use ***Heron's Formula***.
3. Another way to find the area of the triangle is given by the formula: $\boldsymbol{A = \frac{a^2\sqrt{3}}{4}}$, where ***a*** is the ***side*** of the triangle, provided that the ***triangle is equilateral***. It is given that ***AB* = *AC***, but it is not given whether the third side is also equal to the first two. Also, the length of the congruent sides are not given so **there's no way to solve for the area of the triangle using any of the formulas mentioned**.

The answer is **D**.

86. **If a trapezoid has an area of 72 square meters, one base 10 m long, and another base 8 m long, what is the length, in meters, of its altitude?**

A. 4
B. 8
C. 12
D. 16

The ***area of a trapezoid*** is given by the formula below:



Manipulating the formula and solving for the ***altitude***, ***h***, we have:

$$h = \frac{2A}{a+b}$$

$$\boldsymbol{h = \frac{2(72)}{10+8} = \frac{144}{18} = 8\ m}$$

The answer is **B**.

87. **Mr. Reyes has 32 feet of fencing to make a mini-garden area at his backyard. What should the dimensions of the rectangular region be in order to produce the largest possible area for the mini-garden?**

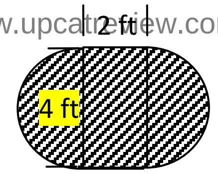
A. 6 ft x 10 ft
B. 7 ft x 9 ft
C. 8 ft x 8 ft
D. 7.5 ft x 8.5 ft

Since the ***perimeter is 32 feet***, the ***sum of the length and the width*** of the rectangular region should be ***16 feet*** (***remember that P = 2(L + W)***). The ***4 dimensions*** given in the choices satisfy the given condition (that is, if you add each dimension, the sum is ***16 feet***). So, we just need to multiply the dimensions in each choice and see which one results to greatest product:

A. 6 x 10 = 60
B. 7 x 9 = 63
C. **8 x 8 = 64**
D. 7.5 x 8.5 = 63.75

The answer is **C**.

<u>Additional note</u>: *Every square is a rectangle. Given a square and a rectangle with the same perimeter, the square always has the greater area.*



88. Find the area of the shaded region.

A. 20.56 ft² B. 33.12 ft² C. 56.52 ft² D. 173.04 ft²

The ***area of the shaded region*** is equal to the ***sum of the area of the rectangle and the areas of the 2 semi-circles***. Take note that the ***diameter of the semicircle equals 4 ft***, and thus its ***radius is 2 ft***:

$$A_{shaded} = A_{rectangle} + 2 \cdot A_{semicircle} = (L \times W) + 2 \cdot \left(\frac{\pi r^2}{2}\right) = (4 \times 2) + (3.14 \cdot 2^2) = 8 + 12.56 = 20.56\ ft^2$$

The answer is **A**.

89. What is the perimeter of an equilateral triangle whose area is $9\sqrt{3}$ cm²?

A. 6 cm B. 12 cm C. 18 cm D. 24 cm

As discussed in ***#85***, the ***area of an equilateral triangle*** is given by the formula: $A = \dfrac{a^2\sqrt{3}}{4}$

Solving for the side ***a*** in the formula, we have: $a = \sqrt{\dfrac{4A}{\sqrt{3}}} = \sqrt{\dfrac{4 \cdot 9\sqrt{3}}{\sqrt{3}}} = \sqrt{36} = 6\ cm$

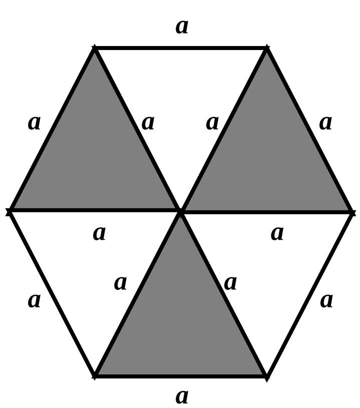
Thus, the ***perimeter*** of the triangle is: ***P = 3a = 3(6) = 18 cm***

The answer is **C**.

90. If the perimeter of a regular hexagon is 24 ft, what is its area?

A. $24\sqrt{3}$ ft² B. $12\sqrt{3}$ ft² C. $9\sqrt{2}$ ft² D. $6\sqrt{2}$ ft²

A ***regular hexagon*** can be divided into ***6 congruent equilateral triangles***:

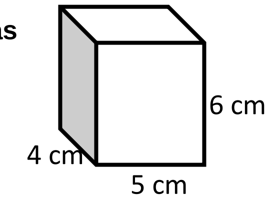


Thus, if the ***perimeter of the hexagon is 24 ft***, its ***side is 24/6 = 4 ft***. It is also the side of each equilateral triangle. Therefore the ***area of the hexagon is 6 times the area of an equilateral triangle***:

$$A_{hexagon} = 6 \cdot A_{equilateral\,\Delta} = 6 \cdot \frac{a^2\sqrt{3}}{4} = 6 \cdot \frac{4^2\sqrt{3}}{4} = 24\sqrt{3}\ ft^2$$

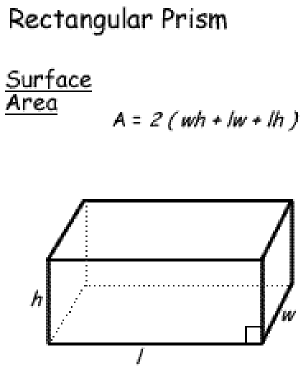
The answer is **A**.

CHAPTER 10: Surface Area and Volume



91. The total surface area of the rectangular box shown, is the sum of the areas of the 6 sides. What is the box's total surface area, in square centimeters?

A. 60 C. 120
B. 74 D. 148

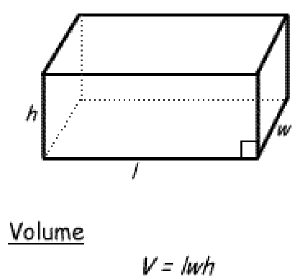


Thus, the box's total surface area
=2(4·6 + 5·4 + 5·6)
= 2(24 + 20 + 30)
= 2(74)
= **148**

The answer is **D**.

92. The volume of a box is 180 cubic in, and the height is 3 in. The length is four times the height. Find the width.

A. 3 in B. 4 in C. 5 in D. 6 in



Solving for the width,

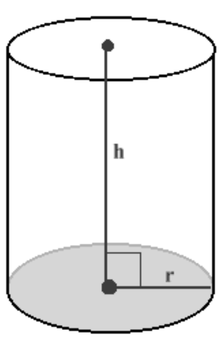
$$w = \frac{V}{lh}$$ where: ***h* = 3, *l* = 4*h* = 4(4) = 12**

Thus, $$w = \frac{V}{lh} = \frac{180}{3 \cdot 12} = \frac{180}{36} = 5$$

The answer is **C**.

93. Find the volume of a right cylinder with base diameter 10 and height 6.

A. 60π B. 150π C. 300π D. 600π



The volume ***V*** of any ***cylinder*** with ***radius r*** and ***height h*** is equal to the ***product of the area of a base and the height***.

<u>**Formula**</u>: **$V = \pi r^2 h$**

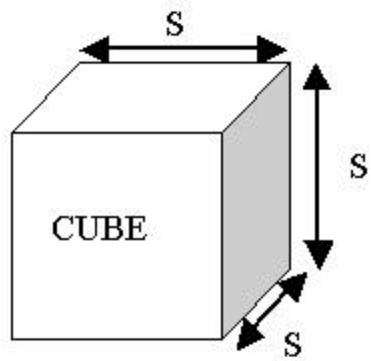
Since the ***base diameter = 10***, ***r = 5***

Thus, **$V = \pi(5)^2(6) = 150\pi$**

The answer is **B**.

94. The volume of a cube is 8000 cm^3. What is the total surface area of the cube?

A. 1200 cm^2 B. 2400 cm^2 C. 3600 cm^2 D. 4800 cm^2



Volume of a cube = s^3

Thus, $s = \sqrt[3]{V}$

$s = \sqrt[3]{8000} = 20\ cm$

Surface Area of a cube =6 s^2

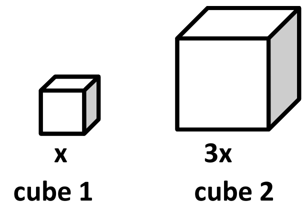
Thus, ***Surface Area =6 (20)^2***

= 2400 cm^2

The answer is **B**.

95. The ratio of the volume of cube 1 to the volume of cube 2 is ____.

A. 1:3 C. 1:9
B. 1:6 D. 1:27



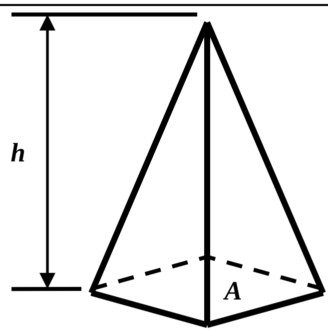
Given the ***side lengths*** of ***two cubes***, the ***ratio of their volumes*** is equal to the ***ratio of their side lengths raised to the third power***:

$$V_1 : V_2 = (s_1 : s_2)^3 = (x : 3x)^3 = x^3 : 27x^3 = 1 : 27$$

The answer is **D**.

96. A pyramid has a volume of 64 ft^3 and a height of 8 ft. Find the base area of the pyramid.

A. 8 ft^2 B. 24 ft^2 C. 256 ft^2 D. 512 ft^2



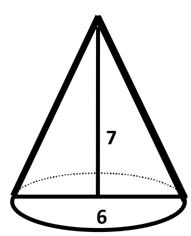
Volume = (1/3)Ah where ***A*** is the area of the base and ***h*** is the perpendicular height

The ***base area*** is thus, $A = \frac{3V}{h}$

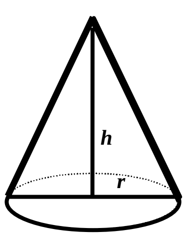
$A = \frac{3(64)}{8} = 24\ ft^2$

The answer is **B**.

97. Find the volume of the cone (see the figure).



A. 21π C. 84π
B. 42π D. 168π

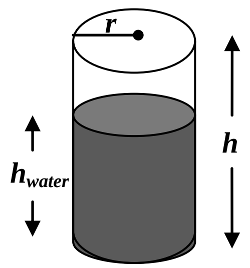


$$V = \frac{1}{3}(\pi r^2 h) = \frac{1}{3}(\pi \cdot 3^2 \cdot 7) = 21\pi$$

The answer is **A**.

98. Three-fourths of a cylindrical can is filled with water. If the diameter and the volume of the can are 12 cm and 360π cm³, respectively, find the height of the water.

A. 10 cm B. 7.5 cm C. 5 cm D. 2.5 cm



Formula: **V = πr²h**

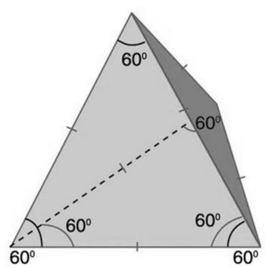
Solving for h, $h = \dfrac{V}{\pi r^2} = \dfrac{360\pi}{\pi \cdot 6^2} = 10\ cm$

Thus, the height of the water is $h_{water} = \frac{3}{4}(10) = 7.5cm$

The answer is **B**.

99. The faces of a triangular pyramid are all equilateral triangles. Find the edge of the pyramid if its total surface area is $100\sqrt{3}$ cm².

A. 10 cm B. $10\sqrt{3}$ cm C. 100 cm D. 400 cm



Total Surface Area (TSA) = 4·Area of one equilateral Δ =

$$4 \cdot \frac{a^2\sqrt{3}}{4} = a^2\sqrt{3}$$

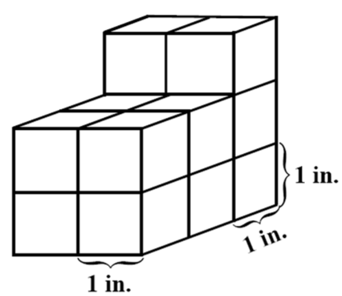
Solving for the edge ***a***, $a = \sqrt{\dfrac{TSA}{\sqrt{3}}} = \sqrt{\dfrac{100\sqrt{3}}{\sqrt{3}}} = \sqrt{100} = 10cm$

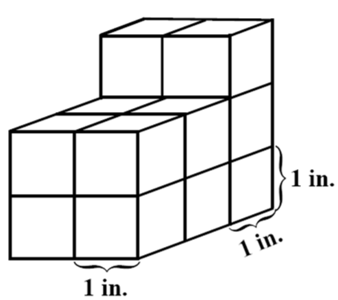
The answer is **A**.

100. One-inch cubes are stacked as shown in the drawing:

What is the total surface area?

A. 19 in.²
B. 29 in.²
C. 32 in.²
D. 38 in.²





We just count the exposed squares:

Right side = 7
Left side = 7
Front = 6
Back = 6
Top = 6
bottom = 6

Total number of squares = 38

The answer is **D**.